Hagen, June 23,2010

Dear Professor Pinkus,

it was a great pleasure for me to meet you personally on the occasion of the CTF-2010 Conference in Sozopol. All the more, as I had been studying before some of your papers and your book on L1-Approximation.

You have kindly suggested that I send you my scanned-in copies of the rare book by Dimitrii Ivanovich Mendeleev (written in Cyrillic Russian and published 1887 by V. Demakov in St. Petersburg)

“Research on Aqueous Solutions According to their Specific Gravity“,

where he puts forward, on page 289, his famous question on the extremal magnitude of the three coefficients of a parabola under the condition that the parabola is bounded by a given constant on a given interval.

This question is followed by a rather long footnote, stretching from page 289 to 290, where D. I. Mendeleev refers six times to Andrei Andreevich Markov whom he had consulted concerning his question. On page 290 there is a second footnote where D. I. Mendeleev refers to A. A. Markov. I have not found these footnotes mentioned in the relevant literature on (the history of) Approximation Theory.

D. I. Mendeleev’s original question and his (first) footnote is reprinted in my paper:

“On V. A. Markov’s and G. Szegö’s Inequality for the Coefficients of Polynomials in One and Several Variables”, 2008
(East J. Approx.14, 319 – 352.
Excerpt: “…seems to be the nucleus of all Markov-type inequalities in Approximation Theory.”).

A. A. Markov in his famous paper:

“On a Question by D. I. Mendeleev”, 1889
(included in Russian original and in an English translation on the HAT-website)

does not refer directly to D. I. Mendeleev’s question (i.e., to p. 289) but rather A. A. Markov refers, on the first page of his paper, to the whole Chapter (§) 86 of D. I. Mendeev’s 1887 book. This Chapter 86 begins on page 287 and ends on page 295.

I therefore include in the attachment the pages 287 – 295, which encompass the Chapter 86, and additionally I include the cover page of D. I. Mendeleev’s book, the foreword page, and the two pages with the table of contents. The file format is JPG.

Please consider if they are suitable for being launched at the Internet website on the History of Approximation Theory (HAT).

Related papers which refer to D. I. Mendeleev's question are

V. A. Markov:

"On Functions Deviating Least from Zero on a Given Interval", 1892
(included in Russian original and in an abridged German translation (Math. Ann. 77 (1916), 213-258) on the HAT website)

V. L. Goncharov:

"The Theory of Best Approximation of Functions", 1945
(included in English translation (J. Approx. Theory 106 (2000), 2 – 57) on the HAT website)

R. P. Boas:

"Inequalities for the Derivatives of Polynomials", 1969
(Math. Mag. 42, 165 – 174.
Excerpt: "Nevertheless, Mendeleev's study led to mathematical problems of great interest, some of which are still inspiring research today".)

R. P. Boas:

"Extremal Problems for Polynomials", 1978
(Am. Math. Mon. 85, 473 – 475, mentioned on the HAT website)

B. Bojanov:

"Markov-type Inequalities for Polynomials and Splines", 2002
(Approximation Theory X: Abstract and Classical Analysis, C.K. Chui, L. L. Schumaker, J. Stoeckler (Eds.), Vanderbilt University Press, Nashville (TN), 31 – 90.

Excerpt: "The story of this subject is more than 100 years old and goes back to the genesis of ApproximationTheory. The beginning was a question, a simple question raised by the famous Russian chemist Dmitrii Ivanovich Mendeleev. In 1887, in his paper "Study of water solutions in terms of their specific gravity", Mendeleev asked: What is the maximum of $|a_k|$, a coefficient of the second degree polynomial $p(x) = a_0 + a_1x + a_2x^2$, provided the deviation from zero of this polynomial on a given interval [a, b] does not exceed a certain fixed number M? This is a great question. It gives rise to a class of interesting and difficult extremal problems".).

Finally, a comment on the two papers by R. P. Boas:

Boas associates Mendeleev's question (which he does not quote literally) to the global problem to determine the largest absolute value of the first derivative of a parabola which is bounded by a given constant on a given interval (i.e., the parabolas vary and the points of the interval vary). This would correspond to the problem 2) on the first page of A. A. Markov's paper and to the second problem, stated on p. 248 (Math. Ann. 77 (1916)), of V. A. Markov's paper.

But Mendeleev's question on p. 289 of his book asks for the local problem to determine the largest coefficients (in absolute value) of a parabola which is bounded by a given constant on a given interval (i.e., the parabolas vary but the point $x = 0$, assuming to lie within the interval considered, is kept fixed, so that the derivatives are taken at $x = 0$ only). This would correspond to the problem 1) on the first page of A. A. Markov's paper and to the first general problem to maximize coefficient functionals, stated on p. 246 (Math. Ann. 77 (1916)), of V. A. Markov's paper, which he has solved, for the special case of single coefficients, on p. 248.

Kind Regards,

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