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In memoriam

## In Memoriam: Bruce L. Chalmers (1938–2015)



Figure 1: Bruce Chalmers.

Bruce Langworthy Chalmers passed away after a decade long battle with Progressive Supranuclear Palsy (PSP), on December 11th, 2015 in Riverside, California. He was 77 years old. He is survived by his wife Patricia (Patti), daughters Cynthia Bartlett and Heather Singarella, sons-in-law John and Paul, and six grandchildren.

Bruce was born on July 29th, 1938 in Dubuque, Iowa. He was a graduate of Philips Exeter Academy, 1956; of Harvard University, B.A. 1960; of Syracuse University, M.S. 1963; and of Stanford University, Ph.D 1967. Bruce's dissertation was written under the direction of Stefan Bergman in the area of several complex variables, and was supported by an IBM fellowship. He joined the faculty of the Department of Mathematics at the University of California, Riverside, in the academic year 1967-68 beginning as Assistant Professor and rising in the ranks to Full Professor, until illness forced him into retirement at the end of the academic year 2007–08.

After a few years of work related to Bergman's Kernel, Bruce has moved into Approximation Theory and Functional Analysis. Altogether he has written more than 70 papers in journals and refereed proceedings. He has the biggest number of papers in JAT by a non editor.

Below is a collection of reminiscences of Bruce by colleagues and friends.

Asuman Aksoy, Claremont McKenna College, Claremont CA (*asuman.aksoy@claremontmckenna.edu*)

I first met Bruce in the 1990s when our mutual colleague, Grzegorz Lewicki, was visiting Claremont from Poland. Later in the year 2000, I spent a semester in UC Riverside, where both our collaboration and friendship began. We worked on questions relating to minimal extensions, where minimality is defined in numerical radius as opposed to in norm. As Bruce did not like to work in his office, much of our joint work was done in informal settings like small diners and Chinese restaurants, where both good food and drink were available. Bruce was both playful and humble towards mathematics. He liked challenges but was not afraid to say "this is too hard for me". He loved life and was a down-to-earth person whose presence was always positive. He had an impressive background but was not boastful about it. He approached other mathematicians with empathy and respect. He fought bravely against Progressive Supranuclear Palsy (PSP). Even in his most physically challenged state, he gave signs that he still remembered our joint work on "diagonal extremal pairs" and himself as a mathematician. Bruce will be sorely missed by his friends, family, and colleagues.

John De Pillis, University of California, Riverside CA (johndepillis@earthlink.net)

During the earliest part of my career as a mathematician at the University of California, Riverside, Bruce had joined the faculty. We had common interests in the areas of analysis and functional analysis so we spent some time together discussing mathematics.

But mathematics, a very human endeavor, is nurtured by interpersonal relationships. And I am proud to add that Bruce and I shared a very harmonious personal rapport.

Are we speaking of mathematical achievement when we discuss billiards? Maybe so. But one of the most enriching and entertaining aspects of my relationship with Bruce was revealed when we played billiards. Let me explain.

Bruce was good at billiards...no he was excellent. There was a time when I would regularly visit his home on Saturday mornings to play the game on his home table. Oh he would let me win on occasion, but it was clear that I would only be honing my skills at best.



Figure 2: Bruce Chalmers as seen by John De Pillis.

**Dany Leviatan**, Tel Aviv University, Israel (*leviatan@post.tau.ac.il*)

I first met Bruce in August or September 1974 at a conference in Canada. That is, more than 41 years ago. I saw a happy family, Bruce, his beautiful wife Patti and two pretty girls, Cynthia and Heather, who since, have grown to be beautiful young women. Bruce and I hit it right away and became good friends. He visited me in Israel a couple of times and I visited him in Riverside many more times. Bruce was a cheerful person, always had a smile on his face. I connect immediately to persons like this and, apparently, Bruce had the same feeling towards me. We always had good time together and, although we worked in different areas within approximation theory, we always enjoyed discussing our interests with one another. Bruce was a very generous man, in more than one way, and I will illustrate it with one example. I once came to Riverside as a visiting professor for a semester and was given to teach two sessions of ODE (as a visitor you do not expect to be given an advanced elective). Bruce who at the same semester was supposed to teach an elective in approximation theory, generously offered me to exchange one session of the ODE course with his approximation theory course. I, of course, declined for two reasons. First, I did not want to take from Bruce a course, that was the place to make the more advanced students interested in him as a future thesis advisor. Second, I was not interested in such students, being a visitor who will disappear shortly after, and did not want to invest in preparing a course in approximation theory, while preparing two sessions of the same course in ODE was like preparing one.

Once we were at a conference in Edmonton, Canada. Bruce, Boris Shekhtman and I were returning to California and had to clear immigration in Edmonton. Boris and I had our passports but Bruce did not have his (that was before Americans had to travel to Canada with passports, and their accent together with a driver's license were enough to prove they were Americans). The immigration office asked Bruce where was he born, to which Bruce answered Dubuque, Iowa. The officer, who had no idea where Dubuque, Iowa, was, or perhaps even doubted that a place like this existed, was quite suspicious at a guy traveling with two untrustful characters (Boris, a Russian immigrant, and me, a Israeli), and would not let him in. It took a while for Bruce to manage to convince the immigration authorities that he was an American citizen, born in the USA.

Eventually, although we worked on different subjects, our interests converged and we even ended up collaborating on a couple of papers, on shape preserving projections, with Mike Prophet.

I will miss Bruce very much, I always thought of him as one of my very few best friends.

**Grzegorz Lewicki**, Jagiellonian University, Kraków, Poland (*Grzegorz.Lewicki@im.uj.edu.pl*)

I first met Bruce in 1995 during my first visit in United States in a bit unusual circumstances. I had been familiar with his name before, because of his papers on minimal projections. Also at that time I had applied for the Fulbright Grant, and E. W. Cheney, who was the top expert in minimal projections, advised me, in case I am successful, to visit Riverside and collaborate with Professor Bruce Chalmers. In 1995 I was invited by Asuman Aksoy for a short visit to Claremont McKenna College, and one day I looked at the map of United States and saw that Claremont was very close to Riverside. So I wrote an e-mail to Professor Bruce Chalmers asking him if we could meet personally. He answered immediately and invited me to give a talk at the Department of Mathematics, University of California, Riverside. One day before my flight to USA I got a

message from the Fulbright Foundation informing me that I had obtained the Fulbright Grant for the academic year 1996–97!! After having finished my talk in Riverside, I told Professor Chalmers about my great success. He was very happy about that.

I spent the whole academic year 1996–97 in Riverside. After one week of my stay in Riverside in 1996, Professor Bruce Chalmers became Bruce, and it was the beginning of our great friendship. Since that time I visited him many times in Riverside (the last time in 2009) and he visited me many times in Kraków. I loved his enthusiasm and passion for mathematics, specially for minimal projections. Also he helped me a lot with his optimistic attitude. Many times when I felt depressed because of lack of progress on our scientific problems, he said to me "Tomorrow we will get it!". We never thought together about mathematics. Usually we worked separately and we met in the afternoon in a pub or small restaurant to discuss our progress, and also more general subjects connected with human life. I would like to add, that my English is far away from being perfect but it was not an obstacle for us to understand perfectly each other. It is worth mentioning that Bruce taught me how to use computers to get theoretical results in mathematics. He believed in computer results (perhaps too much), but in general his predictions were correct. In particular, thanks to his computer experiments, we were able to find some formulas for minimal projections onto finite-dimensional symmetric subspaces of  $l_1$  as well as to find a three-dimensional subspace of  $l_{\infty}^{(5)}$  with maximal absolute projection constants and to prove the Grünbaum "4/3" conjecture in a paper published in Studia Mathematica in 2010. By the way, it was the last Bruce's paper.

I would like to add that Bruce was always very friendly for me. Usually if he noticed that I feel alone in USA or that I have some problems he tried to help me. Also frequently he picked me up at the airport in the middle of the night or in early morning, and he never complained about that. Many times I was invited to his home, usually to watch basketball games.

Also I liked Bruce's sense of humor. For example, some years ago I picked Bruce up at the Kraków airport. He was flying from Chicago with many Polish people for whom it was the first visit to Poland after many years. Many of them were met at the airport with bouquets of red roses. When Bruce came passing through the custom service I said "Hi, how are you doing?". He did not answer to me in a typical way, instead he asked "Where are my roses?". I answered "You will get them next time". A year later, I was better prepared and bought for Bruce a bouquet of five red roses. Again, when I saw Bruce passing through the custom service I said to him "Hi, how are you doing?". This time he started a conversation with a question "Why only five?".

Unfortunately, I had to unexpectedly quit my scientific collaboration with Bruce in 2008 because of Bruce's illness. I hope that we will meet in our second life and we will work together on our favorite mathematical problems.

**Frederic T. Metcalf**, University of California, Riverside CA (*fredtmetcalf@gmail.com*)

I was involved in the hiring of Bruce Chalmers at UCR in 1967, and was a co-author with Bruce on 11 papers spread over many decades. Bruce was not only a colleague, but a friend I will miss.

Bruce was especially noted for his infectious *enthusiasm*, as well as a *tenacity* that kept both him and his many co-authors<sup>3</sup> working on projects which, in some cases, were very long-term. In light of these prominent characteristics I will relate the first, and longest, of Bruce's attacks on a difficult and outstanding problem.

Following several years of continuing work related to the Bergman kernel and his dissertation, Bruce became interested in interpolation issues. This lead to his attending in 1972 one of the Texas conferences on approximation theory. Upon his return to UCR, Bruce was excited to report that the conference had been abuzz with talk of an unsolved problem, namely, the determination of a projection from C[-1, 1] to the quadratics having the least possible norm. Bruce had an idea on how to solve this problem—but it just needed some serious computing, and so he brought me in to assist in the quest for a minimal projection onto the quadratics. While Bruce's original idea did not work out, his tenacity kept our team of two working on the project for the next 20 years. Roughly eight years were spent on determining the form of the minimal projection (see [1] below), another eight years to develop a proof of minimality (see [2] below), and a final four years to get the results published.

That this problem is non-trivial is clear from the structure of the minimal projection from C[-1, 1] onto the quadratics (see [1] below):

$$\begin{aligned} (P_{\min}f)(x) &= (A - Cx + Dx^2)f(-1) + B(1 - x^2)f(0) + (A + Cx + Dx^2)f(1) \\ &+ \sum_{k=1}^2 \left( \int_{-s_{k2}}^{-s_{k1}} + \int_{s_{k1}}^{s_{k2}} \right) \frac{(b_k + a_k|s|) + (c_ks)x + (-b_k + d_k|s|)x^2}{(1 + w_k|s|)^3} f(s) \, ds, \end{aligned}$$

for  $f \in C[-1, 1]$ . The values of the parameters may be determined giving  $||P_{\min}|| = 1.220173...$  (this may be compared with the interpolation projection based on the points  $\{-1, 0, +1\}$  which has a norm of 1.25). Unmentioned here is the great benefit of this determination: an understanding of minimal projections in much more general settings.

[1] B. L. Chalmers and F. T. Metcalf, "Determination of a minimal projection from C[-1, 1] onto the quadratics", Numerical Functional Analysis and Optimization, Vol. 11 (1990), p. 1–10. [2] B. L. Chalmers and F. T. Metcalf, "The determination of minimal projections and extensions in  $L^{1}$ ", Trans. Amer. Math. Soc., Vol. 329, No. 1 (1992), p. 289–305.

Let me add a list of Bruce's co-authors (with a count of multiplicity):

Aksoy, A. (1)	Franchetti, C. (1)	Mason, J. (2)	Prophet, M. (6)
Bani, M. (1)	Giaquinta, M. (1)	Metcalf, F. T. (11)	Ribando, J. (1)
Bergman, S. (1)	Johnson, D. (1)	Mupasiri, D. (1)	Shekhtman, B. (8)
Egger, A. (1)	Leviatan, D. (1)	Ostrowskii, M. (2)	Taylor, G. (7)
Ferguson, L. (1)	Lewicki, G. (5)	Phillips, G. M. (1)	Taylor, P. J. (1)

**Michael Prophet**, University of Northern Iowa, Cedar Falls IA (*prophet@math.uni.edu*)

Bruce had a unique relationship with computer technology, somehow being simultaneously interested and apathetic. He would declare that having a computer nearby "raises my IQ by at least 10 points!" but then found no use for the attached mouse. It was not until the early 90s that I (perhaps) finally convinced him of the utility of using a mouse...

For several years we were in continuous email contact and he never deleted a single message. He did not delete email from anyone. And, to add to the enormity of this problem, he refused to create any organizational folders within his InBox. When he needed to find a previously received note, he would visually scan through the massive array of saved messages, slowing hitting his PageDown key. He claimed that this scanning practice was an excellent cerebral exercise. But Bruce also knew how to 'automate' a scanning process and this would prove quite valuable in his final research effort.

During the late 80s Bruce and colleagues (Fred Metcalf and Boris Shekhtman) tackled the Grunbaum (or 4/3's) Conjecture and submitted it for review in January 1992. But there were referee questions that could not be resolved and, as such, another group published a paper during 1992 containing a 4/3's proof (as well as some generalizations). I think it is accurate to say that Bruce was quite disappointed.

Going through this paper, Bruce discovered (in cases beyond the 4/3's setting) numeric inconsistencies with his computer-generated results. And that is all he needed to see. He launched himself into the task of building an enormous set of (Mathematica) code that "scanned" through millions upon millions of possible configurations, to illustrate the published paper must have an error.

For the next decade, Bruce collaborated frequently with Greg Lewicki on minimal projection problems. But during that entire time Bruce would not "let go" of the published 4/3's proof. Poor Greg was continually pushed by Bruce to "find the mathematical error" in the paper. I distinctly remember a conversation I had with Greg in 2005 while he was visiting me prior to going to Riverside. He declared that he could not take it anymore and would, just to appease Bruce, go through that paper line by line to show Bruce, once and for all, that everything was correct and the error must reside in his scanning code.

And, of course, I will never forget the email from Greg when he reported that, indeed, he had found a (small but significant) error in the paper.

The Chalmers–Lewicki proof of the 4/3's conjecture was accepted for publication in 2010. It was Bruce's final paper.

**Boris Shekhtman**, University of South Florida, Tampa FL (*shekhtma@usf.edu*)

We met in Riverside and bonded over "minimal projections." Bruce was obsessed with the subject. Never before (or since) have I experienced that much enthusiasm and inventiveness, bordering on crazy, as I did working with him.

It is hard to pinpoint the moment when we became friends, but when we did, I found that he had the same attitudes toward everything in life. I feel very lucky. He was and is my best friend. The kind of friend that every guy is fortunate to have: a loving, generous, non-judgmental eternal optimist. He once told me, Life is a party the best news is: we are all invited, and this may be the best summation of Bruce.

We traveled, played tennis, spent time in bars talking, everything was on the menu from mathematics to the meaning of life. We invented a Mai Tai transform of Banach spaces in a Chinese restaurant, and a rusty nail personality in Franks bar. We had fun at that party called life.

Bruce was passionate in his believes and true to his personality, effortlessly rejecting any social pressures to change it. A student once wrote in his evaluation: Doctor Chalmers is the best. Every woman wants him; every man wants to be him. I could never be him, but what little I borrowed made me a better man and a better mathematician.

Bruce, thank you for your friendship; I miss you buddy. I am not certain where you are now but I am certain that if there is a party there you are invited.

**G. D. (Jerry) Taylor**, Colorado State University, Fort Collins CO (*taylor.jerry@gmail.com*)

Bruce and I had a common interest in approximation theory and this led to joint work and a continuing friendship. I first met Bruce when he came to the Symposia on Approximation

Theory and Applications, Michigan State University, 22–24 March, 1972, that I organized. This occasion was followed with my attending the 1972 summer NSF regional approximation theory conference featuring G. G. Lorentz organized by Bruce, Fred Metcalf and Vic Shapiro as I remember. It was here that my first joint effort with Bruce and others occurred with a paper studying the rank of Hermite–Birkhoff interpolation (Bruce, D. Johnson, F. Metcalf and I). In 1977, P. L. Butzer invited me to submit an enlarged survey updating an earlier 1973 survey I had published on uniform approximation with constraints (for Jber.d. Dt. Math. Verein). I was well aware of Bruce's work in this area providing a functional analytic general treatment. Thus, it seemed natural to me to ask him to join me in this endeavor. He agreed and this led to our survey of the state of uniform approximation with constraints in 1979 containing some 180 references. In the succeeding years we found additional topics of common interest resulting in five additional publications, three of which were concerned with the concept of strong unicity in the presence of constraints.

Personally, I particularly enjoyed Bruce's wit and the many stories he told. All these stories always presented others in a positive light. On one of his visits to Colorado State University, we went for a hike in Rocky Mountain National Park. Since he had just come from sea level, hiking from Bear Lake (9600 ft.) to the top of Flattop Mountain (12,234 ft.) was a bit of a challenge and he was up to it. But as he related to others he only managed because I had a backpack full of treats and kept rewarding him as we hiked. I also remember Bruce as being ahead of the times in that he sported a gold necklace in the 70s. Since it was a chaotic mixture of different sized letters, I asked him if it spelled something? His answer Outrageous. One comment that Bruce frequently repeated about his mathematical career was I cannot believe that I can get paid to do something that I enjoy so much. (I agree!) This is the Bruce I enjoyed and will miss.

## Publications of Bruce Chalmers in J. Approx. Theory

J. Approx. Theory, Volume 4, Number 2, June 1971 Bruce L. Chalmers and Le Baron O. Ferguson Sets of best approximation in certain classes of normed spaces 194–203.

J. Approx. Theory, Volume 7, Number 3, March 1973 Bruce L. Chalmers Uniqueness of best approximation of a function and its derivatives 213–225.

J. Approx. Theory, Volume 7, Number 4, April 1973 Bruce L. Chalmers On weak underpolynomials of generalized infrapolynomials 391–394.

J. Approx. Theory, Volume 20, Number 3, July 1977 B. L. Chalmers and F. T. Metcalf Minimal generalized interpolation projections 302–313.

J. Approx. Theory, Volume 32, Number 3, July 1981 Bruce L. Chalmers A natural simple projection with norm  $\leq \sqrt{n}$  226–232.

J. Approx. Theory, Volume 37, Number 1, January 1983 B. L. Chalmers and G. D. Taylor A unified theory of strong uniqueness in uniform approximation with constraints 29–43.

J. Approx. Theory, Volume 37, Number 3, March 1983 B. L. Chalmers, F. T. Metcalf, and G. D. Taylor Strong unicity of arbitrary rate 238–245.

J. Approx. Theory, Volume 37, Number 4, April 1983 B. L. Chalmers, A. G. Egger, and G. D. Taylor Convex  $L^p$  approximation 326–334.

J. Approx. Theory, Volume 40, Number 4, April 1984 B. L. Chalmers and J. C. Mason Minimal  $L_p$  projections by Fourier, Taylor, and Laurent series 293–297.

J. Approx. Theory, Volume 41, Number 3, July 1984 B. L. Chalmers, E. H. Kaufman Jr., D. J. Leeming, and G. D. Taylor Uniform reciprocal approximation subject to linear constraints 201–216.

J. Approx. Theory, Volume 42, Number 2, October 1984 Mohamed S. Bani and Bruce L. Chalmers Best approximation in  $L^{\infty}$  via iterative Hilbert space procedures 173–180.

J. Approx. Theory, Volume 48, Number 2, October 1986 Bruce L. Chalmers The Fourier projection is minimal for regular polyhedral spaces 201–206.

J. Approx. Theory, Volume 53, Number 3, June 1988 B. L. Chalmers, G. M. Phillips, and P. J. Taylor Polynomial approximation using projections whose kernels contain the Chebyshev polynomials 321–334.

J. Approx. Theory, Volume 98, Number 2, June 1999 B. L. Chalmers, D. Leviatan, and M. P. Prophet Optimal interpolating spaces preserving shape 354–373.

J. Approx. Theory, Volume 138, Number 2, February 2006 B. L. Chalmers, D. Mupasiri, and M. P. Prophet A characterization and equations for minimal shape-preserving projections 184–196.