In Memoriam Borislav D. Bojanov (1944–2009)



Friends and colleagues,

Academician Borislav Bojanov passed away suddenly on April 8, 2009. There are no words to describe the shock and overwhelming sadness that all of us, his friends, colleagues and students felt. It is so sad and unjust that a man of such talent, such capability, kindness and intelligence should now be so suddenly spirited away at the age of only sixty four, before he had achieved many of the things of which he was capable.

IN MEMORIAM

Professor Bojanov was a world-known mathematician, author of more than 130 papers, published in peer reviewed journals. His research spanned over a vast variety of topics, mainly in the area of Approximation Theory, Classical and Numerical Analysis. A common feature of his work was its beauty, the elegant arguments and deep insight, which had allowed him to not only solve particular mathematical problems, but also to find far reaching extensions and generalizations.

A substantial part of Professor Bojanov's research was devoted to problems of Optimal Recovery. Both his Ph. D. and Dr. Sci. Theses, entitled "Optimal Interpolatory and Quadrature Formulas" and "Optimal Recovery of Functions and Functionals", respectively, treat problems of finding the best approximation to functions (or functionals) from a given class when some partial information for these functions is available. Although Academician Blagovest Sendov was his PhD advisor, Professor Bojanov's interest to Optimal Recovery was first inspired by the discussions he had with Professor Nikolai Sergeevich Bakhvalov during his ten month visit to Moscow State University in the early seventies.

One of his first contributions to the theory of optimal recovery is the proof of existence, characterization and sharp error asymptotic of the optimal quadrature formula with preassigned multiplicities of the nodes for functions from the Hardy spaces H^p . He also solved completely the fundamental problem of existence, uniqueness and characterization of optimal quadrature formulas with multiple nodes for functions from the Sobolev classes. In particular, he proved that the optimal quadrature formula for functions from the periodic Sobolev classes must have equally spaced nodes, a fact that was conjectured long before Bojanov's contribution and resisted the attempts of many outstanding mathematicians.

The problems of optimal recovery methods for functions in the Sobolev classes are closely related to the investigation of specific spline-functions. Among Professor Bojanov's essential contributions to the theory of spline-functions are his results on monosplines of least deviation, *B*-splines and σ -perfect splines with Birkhoff knots, total positivity of the truncated power kernel in the general setting of Birkhoff type data, Markov interlacing property for splines and perfect splines, and others. Jointly with H. Hakopian and A. Sahakian, he wrote a monograph on spline functions.

Establishing existence and uniqueness of optimal recovery methods is a fairly difficult task. The attempt to obtain the best scheme may lead to a method which uses type of information, different from the initially declared one. As a by-product of settling the existence and uniqueness problem, Bojanov proved important comparison theorems for recovery schemes based on various type of information of the same cardinality.

Along with optimal methods of integration, Professor Bojanov investigated quadrature and cubature formulas that use non-standard type of information and are exact for the largest subspace of a given linear space (Gaussian quadrature and cubature formulas). For example, he proved the existence and uniqueness of the generalized Gaussian quadrature for extended and for weak Tchebycheff systems, the existence of a quadrature formula with double algebraic degree of precision which uses Birkhoff type information about the integrand, the existence and uniqueness of a Gaussian "interval" quadrature that is based on linear combinations of integrals of the integrand over non-overlapping intervals with prescribed lengths. He showed the existence and uniqueness of a cubature formula for the unit ball in \mathbb{R}^m , which uses n integrals of the integrand over hyperplanes (Radon projections) and is exact for all polynomials in m variables of total degree 2n - 1, the existence and uniqueness of the integrand or certain differential operators, and has the highest possible polyharmonic order of precision.

One of Bojanov's favorite research topics was polynomial inequalities. He proved an old-standing conjecture of P. Erdös about the "longest" polynomial. In 1939, Erdös himself had shown that $\cos nt$ and its shifts are the only trigonometric polynomials with maximal length of their graphs among all trigonometric polynomials of order n whose uniform norm is bounded by 1 in $[0, 2\pi]$. He conjectured that the "longest" algebraic polynomial of degree n with norm not exceeding 1 in the interval [-1, 1] should be $\pm T_n$, where T_n is the n-th Tchebycheff polynomial of first kind. Professor Bojanov gave a brilliant proof to this conjecture more than forty years after it was formulated. Simultaneously, he established a beautiful extension of the classical Markov inequality, showing that

$$||f'||_p \le ||T'_n||_p ||f||$$

holds for every polynomial of degree n, where $\|\cdot\|$ and $\|\cdot\|_p$ are the C[-1,1]and the $L^p[-1,1]$ norms, respectively, and $1 \leq p \leq \infty$. Both proofs are based on the observation that some functionals of polynomials are monotonic with respect to the quantities $h_k = |f(\xi_k)|$ that are the absolute values of the local extrema of the oscillating polynomial itself. It turns out that the length and the L^p norms of the derivative of the polynomial are such monotonic functionals. Professor Bojanov extended and refined the beautiful idea of the monotonicity with respect to the h's to obtain Markov and Turán type polynomial inequalities for algebraic polynomials with multiple zeros, for oscillating trigonometric polynomials and perfect splines. Recently, he obtained jointly with his student N. Naidenov a proof of another Erdös' conjecture, the one concerning the maximal length of trigonometric polynomials on arbitrary interval.

Professor Bojanov's refined taste for beautiful problems determined his work on several classical open problems. One of them is the famous Sendov conjecture about the critical points of complex algebraic polynomials. He proved that the conjecture is true at least "asymptotically". Various extremal problems for polynomials in the complex plane pertaining to Sendov's and Smale's conjectures are formulated in Bojanov's last written paper.

Bojanov was among the first mathematicians to apply the topological degree of a mapping to prove existence and uniqueness of solutions to various problems in Approximation Theory. He also worked on bivariate polynomials of least deviation from zero, moment preserving approximations, best one-sided L_1 -approximations by blending functions, interpolation by bivariate polynomials, Landau-Kolmogorov type inequalities and others. Some of his papers represent new, alternative insight to classical results in Approximation Theory, such as the Hobby-Rice and Gauss-Krein theorems, the Jackson theorem and its extension to Tchebycheff systems, the Remez inequality, the Bernstein comparison theorem for best uniform approximation of smooth functions and others. Recently, he suggested alternative proofs to the classical V. Markov inequality, as well as to some of its generalizations given by R. Duffin and A. Schaeffer.

Being a world class internationally recognized researcher, Borislav Bojanov had been invited to give lectures and shared his experience at various research centers and universities all over the world. He was an invited speaker at many prestigious mathematical forums and conferences, among which was the First European Congress of Mathematics in Paris in 1992.

Professor Bojanov served for fifteen years at the Editorial Board of Journal of Approximation Theory. He was the founder of East Journal on Approximations and the journal's Editor in Chief.

Professor Bojanov supervised ten PhD students and affected the scientific life of many others. Nowadays, his students have successful careers in the academia and the financial industry in Bulgaria, USA, Canada, Brazil and the United Kingdom. We, his students, commemorate him with respect and gratitude for all that he has given us.

Professor Bojanov was a great human being, a fine mathematician and an incredible Teacher. He lived a remarkable life, one that inspired us, his students, greatly. His attitude, broad range of interests and happy demeanor made him a wonderful person to know. He was patient, and generous with his time and affection, always ready to help and guide us through our professional journey. He was not only our Teacher and showed us how to become mathematicians, but he was like a father who taught us to be honest, reliable, persistent, and how to face the challenges of life at any level. Professor Bojanov was philosophical in his approach to life. He had a great perspective, especially when it came to little things, never displaying anger or impatience. Instead, he showed a great dignity and humour. Even when he was critical of somebody or something, he managed to show it in a unique, delicate way, with a joke or interesting story. We will miss his perspective and his gentle humor, the depth and scope of his knowledge, the warmth he extended to everyone he met. We will remember his friendly smile, his endless energy, his encouraging words and his pride of being a Bulgarian. Professor Bojanov always talked with such excitement about our glorious past and history, about the numerous achievements of the bulgarians abroad, about the quality and the potential of our students and our nation. He was well aware of the needs of our young generation and was the first to initiate a movement for the declaration of the education as a national priority for Bulgaria. His students were always his first concern. During his service as a Dean, he used the entire bonus for his Dean position to create Fellowships for talented students with financial difficulties. We became aware of his generosity only after he passed away, when some of the few people who knew revealed it. He had organized the payment of the stipends in such a way that the students didn't know the source of the money. That was him, the person Borislav Bojanov: generous, noble, delicate, modest, with strong beliefs, hopes and ideals.

Professor Bojanov pursued his many endeavours diligently, and always rose to meet a challenge. As a Dean of the Department of Mathematics and Informatics at Sofia University, he raised money to renovate and extend the department building. A whole new floor, new auditoriums, offices, apartments for visitors, elevator and a ramp for handicapped students were added to the building, which became the first education building in Bulgaria with built-in access for students with special needs. He also personally participated in the remodeling of the park around the building. It now has a completely new look with its gardens, alleys, ornamental clock, chess corner and chapel and is a nice secluded spot for not only the students but also for the people living in the neighborhood.

On the scientific side, Professor Bojanov maintained and broadened the research activity at the Department. He revived the departmental Colloquium and organized the international conference "Pioneers of Bulgarian Mathematics", dedicated to two of the most prominent Bulgarian mathematicians, Academicians N. Obreshkov and L. Tchakaloff. Four BS programs and more than fifteen MS programs were created under his guidance. Borislav Bojanov was an extremely successful Dean whose contributions to the department earned the respect, gratitude and love of his colleagues. We include only a few comments from his colleagues, friends and former students:

"... He was always encouraging the young mathematicians and helping them in any possible way - from the formulation of a problem to work on, to technical details in the manuscript preparation, no matter whether he is a co-author or not. ... It is true what people say that it is not possible for a man to be a great scientist without being a great human being. Professor Borislav Bojanov was very friendly and accessible man, whose good mood and desire to work was contagious. We lost a great scientist and human being."

(N. Naidenov)

IN MEMORIAM

"I was fortunate to work under the supervision of this extraordinary mathematician and wonderful man. His nice character, enthusiasm, love for his work and his spirit are a model for everybody who knew him. ... My words are powerless to express my gratitude for Professor Bojanov."

(I. Georgieva)

"I am so sorry... I was terryfied when I opened the e-mail with this news. Whatever I say it will not be enough. We, who were close to him, know what a wonderful person he was. I just can not believe it. A person is powerless in front of death, but I am sure that, besides his family, there are a lot of other people who sincerely grieve for him. We, his students, are among them. I am also sure that he will always be with us in a certain way..."

"... I lost a Teacher and a Friend ..."

(D. Dryanov)

(P. Petrov)

"... Whatever one could write about Professor Bojanov, it will not be enough to entirely describe his personality and contributions. His most important qualities, I think, are his enormous love for Bulgaria, his outstanding contributions in mathematics, his unforgettable teaching ability, his students, and the fact that he was a wonderful man. Thank you, my Teacher!"

(L. Milev)

"Nothing can fill in the huge emptiness in our lives after the death of Professor Bojanov. Maybe we still can not and do not want to accept that He is not anymore among us. I can not find words to thank him for his help, support, trust and faith in me, for the good words and encouragements, for the active environment and possibilities to meet world class scientists, for everything ... Thank you!"

(V. Gushev)

"His spirit will always be with us..."

(R. Uluchev)

"... Borislav was a great patriot and was working hard for the benefit of our country. As Dean of the Faculty of Mathematics and Informatics of Sofia University, Professor Borislav Bojanov made significant progress in improving the conditions for education in a difficult political and economic environment. His creation of the East Journal on Approximations is an example of his drive to promote Bulgaria as an important country in mathematics."

(Bl. Sendov)

"... A great mathematician, a kind person..."

(M. Vianello)

"... He was a great mathematician and remarkable man."

(Vl. Babenko)

"... I was shocked and deeply saddened by the sad news. ... He was a good friend and a great mathematician."

(Y. Xu)

"... It is a big loss for all of us and for the whole academic association, not only mathematical!"

(G. Milovanovic)

"... Indeed it is a big loss for many people all over the world."

(B. Kashin)

"... This is a tremendously sad news, indeed. Borislav was a great mathematician, and a very warm person, our friend!"

(K. Oskolkov)

"... I am keeping him in my mind as a great mathematician and a great friend of all mathematicians who had the chance to meet him."

(W. Plesniak)

"... This is terrible loss for the FAMILY, for the MATHEMATICAL FAMILY and for his FRIENDS. This is also serious loss for the SCIENCE." (Z. Ciesielski)

"... That is indeed very, very sad news. Borislav and I were good friends even though we did not see each other often enough, especially these last few years."

(A. Pinkus)

"... It is really a shock and a big loss."

(S. Konyagin)

"... I am so shocked I do not know what to write. We are going to live in another world - world without Borislav."

(H. Hakopian)

"... Borislav was an excellent mathematician, powerful administrator, unusually fine person. He was a friend. It is sad when friends pass away." (V. Arestov)

"... I am so sorry, and I don't know what to say. My sincere condolences to his family (which unfortunately I did not know) and to his extended scientific family (which I had and will always have a privilege to be a part of). I am sharing your pain."

(A. Shadrin)

"... He was a great friend and fine mathematician. All of us enjoyed his remarkable vitalistic attitude. Last year I met Borislav twice and I was very impressed..."

(F. Marcellan)

" \ldots I am shocked by the terrible news \ldots I liked him as a person and as a mathematician."

(N. Dyn)

"... I myself considered Borislav my no.1 friend among all mathematicians whom I got acquinted during my many travels everywhere in the world. Somehow he was such an open, friendly, warm-hearted, vigorous, supportive, nice personality, that I always felt that I can count on him. Now it all belongs to the past ..."

(S. Revesz)

"... I am shocked and so sorry ... It is really terribly sad. ... I still cannot believe that Borka passed away."

(H. Wozniakowski)

"The unexpected, premature and shocking death of acad. Borislav Bojanov - Borkata deeply saddened many of his friends, colleagues and students from Bulgaria and all over the world! I lost a real friend, the profession lost an amazing mathematician, and our country lost a patriot and a great Bulgarian.

A man with exceptional values, a citizen with deep understanding of the fate of our people and country, a scientist with extraordinary contributions, a colleague with high moral and principles and a friend with a rare sense of responsibility – all this was the person Borislav Bojanov – one of the most prominent Bulgarians of our time.

Borkata will stay forever in our hearts, and his endless humor, constant optimism, cheerful character and wisdom will warm our souls in good times and will support us during the challenges of life."

(R. Lazarov)

We all will miss our Teacher, colleague and friend dearly. But we will treasure his memory forever. May he rest in peace.

On behalf of his students, friends and colleagues,

Dimitar Dimitrov Geno Nikolov Guergana Petrova

CURRICULUM VITAE \mathbf{of} Borislav Bojanov

PERSONAL DATA

Birth Place: Shemshevo, Bulgaria Birth Date: 18 November, 1944 Wife: Rositza Sons: Yavor, Tchavdar, and Tihomir Grandsons: Kalin and Nikola

EDUCATION

1975	University of Sofia, Ph. D. in Mathematics
	Thesis: Optimal Interpolation and Quadrature Formulae
	Supervisor: Acad. Blagovest Sendov
1965 - 1968	Wroclaw University, Wroclaw, POLAND, M. S.
	Supervisor: Prof. Stefan Paszkovski
1962 - 1965	University of Sofia, Sofia, BULGARIA

ACADEMIC CAREER

1985 - 2009	University of Sofia, Sofia. Professor of Mathematics
2003 - 2007	Dean of the Faculty of Mathematics and Informatics
1977 - 1985	University of Sofia, Sofia. Associate Professor
1970 - 1977	University of Sofia, Sofia. Assistant Professor
1968 - 1970	University of Sofia, Sofia, Instructor

HONORS

1992, Invited speaker in The First European Congress of Mathematicians, Paris

1973, Union of the Balkan Mathematicians, Award for young researchers, for outstanding research achievement

Ph.D. STUDENTS

Georgi Grozev (1987) Rumen Uluchev (1990) Hrisina Draganova (1998) Natasha Dicheva (2001) Nikola Naidenov (2003)

Geno Nikolov (1990) Dimitar Dimitrov (1992) Lozko Milev (2000) Petar Petrov (2001) Irina Georgieva (2005)

JOURNAL EDITOR

East Journal on Approximations, Editor-in-Chief, 1995 – 2009 Journal of Approximation Theory, Elsevier, 1994 – 2009 Numerical Algorithms, Springer, 2007 – 2009 Annuaire l'Université de Sofia, Faculté Phys.-Mathématique, Bulgaria Numerical Mathematics: Theory, Methods and Applications, China Facta Mathematica (Niš), Serbia Studia Universitatis Babes-Bolyai Mathematica, Romania

RESEARCH AND VISITING POSITIONS

2003	Polish Academy of Sciences, Banach Center (2 months)
2002	University of Pau, France (1 month)
2001	Duisburg University, Germany (2 months)
2000	University of Pau, France (1 month)
1999 - 2000	Texas A&M University, Texas (9 months)
1999	University of South Carolina, USA (1 month)
1997	Duisburg University, Germany (3 months)
1997	IBILCE, Sao Jose do Rio Preto, SP, Brazil (2 months)
1994	Polish Academy of Sciences, Poland (2 months)
1994	University of Pau, France (2 months)
1992	Israel Institute of Technology, Technion (1 month)
1989	Université de Montreal, Montreal, Canada (2 months)
1986 - 1987	Texas A&M University, Texas (9 months)
1986	University of Oregon, Eugene, USA (2 months)
1985	Columbia University, New York, USA (1 month)
1984	Université de Montreal, Montreal, Canada (2 months)

1971–1972 Moscow State University (10 months)

SELECTED ADDRESSES

2008 International Conference on Approximation in scientific computing (ICASC'08), Beijing, China, Optimal recovery of bivariate functions and integrals

- 2008 IX International Conference "Approximation and optimization in the Caribbean", San Andres Island, Colombia, Plenary lecture: *Majorization of polynomials on the plane*
- 2007 "Frontier lecturer", Texas A& M University

 Majorization of polynomials on the plane
 Interpolation by bivariate polynomials
 Two famous conjectures about the critical points of polynomials
- 2007 International conference "Extremal problems in complex and real analysis", May 21–27, Moscow, Russia,
 Invited mini course: Extremal problems for oscillating polynomials

2006	First Dolomites workshop on constructive approximation and applications, Alba di Canazei (Trento), Italy, September 8-12, 2006, Plenary speaker: Interpolation by bivariate polynomials
2005	International conference "Multivariate Approximation", September 25 - October 1, 2005, Bommerholtz, Germany, Plenary lecture: Interpolation by bivariate polynomials
2005	International conference "Harmonic Analysis and Approximations, III", September 20–27, Tsahkadzor, Armenia, Plenary lecture: Interpolation by bivariate polynomials based on Radon projections
2005	International conference "Function spaces, Approximation theory, Nonlinear analysis", dedicated to the centennial of Sergei Mikhailovich Nikolskii, May 23–29, 2005, Moscow, Russia, Plenary lecture: <i>Optimal quadrature formulae</i>
2004	Approximation and Probability, International conference on the occasion of the 70th anniversary of Professor Zbigniew Cisielski, 20–24 September, Bendlewo, Poland, Plenary lecture: Interval quadrature formulas of Gaussian type
2004	5-th International Conference on Functional Analysis and Approximation Theory, Maratea, Italy, June 16–23, Plenary lecture: <i>Interpolation by bivariate polynomials</i>
2004	International conference celebrating the 70-th birthday of Laura Gori: Classical and New Approximation Spaces: Theory and Applications, Rome, February 5–7, Invited lecture: <i>Quadrature formulae of non-standard type</i>
2001	Tenth International Conference on Approximation Theory, St. Louis, Missouri, USA, Plenary lecture: Markov-type inequalities for polynomials and splines
2000	International Conference on Multivariate Approximation, Haus Bommerholtz, Germany, Plenary lecture: Integration of Polyharmonic Functions
1996	International memorial conference "D.S. Mitrinovic", Niš, Yugoslavia, Invited lecture: Jackson and Whitney Constants
1995	Conference dedicated to the 90th birthday of S. M. Nikolski, Moscow, Russian Academy of Sciences, <i>Polynomial Inequalities</i>
1994	Conference on Information Based Complexity, Dagshtull, Two Effective Algorithms in the Recovery of Functions
1994	Total Positivity and Applications, International conference, Jaca, Spain, Invited lecture: Total Positivity of the Spline Kernel and its Applications

1993	Paul Erdös Conference on Approximation Theory, Budapest,
	Jackson's Type Theorem for Tchebycheff Systems
1992	First European Congress of Mathematics, Paris,

- Invited lecture: Optimal Recovery of Functions and Integrals
- 1987 Conference on Approximately Solved Problems, Columbia University, New York, USA $\sigma\text{-}Perfect\ splines$
- 1981 Oberwolfach, Germany, Conference "Numerical Integration" Oscillating Polynomials of Least Norm
- 1978 Oberwolfach, Germany, Conference "Numerical Integration" Uniqueness of the Monosplines of Least Deviation

MEMBERSHIPS

1997, Member of the Bulgarian Academy of Sciences

RESEARCH INTERESTS

Approximation Theory, Optimal Recovery, Polynomial Inequalities, Splines, Quadrature Formulae, Interpolation by bivariate polynomials

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List of Publications of Borislav D. Bojanov

Papers

- B. D. Bojanov, On an estimation of the roots of algebraic equations, Zastos. Mat. 11 (1969/1970) 195–205. MR0269814 (42 #4709)
- B. D. Bojanov, Supplement to the paper of Lupas and Müller, Aequationes Math. 5 (1970), 38–39. MR0279496 (43 #5218)
- B. D. Bojanov, V. M. Veselinov, A note on the approximation of functions in an infinite interval by linear positive operators, *Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.)* 14 (1970), 9–13 (1971). MR0324275 (48 #2627)
- B. D. Bojanov, Uniform parametric approximation of |x| by algebraic polynomials, (Bulgarian), Annuaire Univ. Sofia, Fac. Math. 64 (1969/70), 331–337 (1971). MR0303180 (46 #2318)
- B. D. Bojanov, Polynomials of best approximation with respect to Hausdorff distance, (Bulgarian) Annuaire Univ. Sofia, Fac. Math. 64 (1969/70), 161–170 (1971). MR0308654 (46 #7768)
- B. Sendov, B. D. Bojanov, On a property of a class of linear positive operators, *Annuaire Univ. Sofia, Fac. Math.* 64 (1969/70), 115–117 (1971). MR0313684 (47 #2238)
- B. D. Bojanov, The ε-entropy of a certain class of analytic functions, (Russian) Annuaire Univ. Sofia, Fac. Math. 66 (1971/72), 363–370 (1974). MR0396951 (53 #811)
- B. D. Bojanov, V. M. Veselinov, On the approximation of functions by Haar series, *Mathematica (Cluj)* 14 (1972), 189–192. MR0355427 (50 #7901)
- B. D. Bojanov, The relationship between the variation of a function and its modulus of nonmonotonicity, (Bulgarian) Bulgar. Akad. Nauk. Otdel. Mat. Fiz. Nauk. Izv. Mat. Inst. 13 (1972), 99–103. MR0422537 (54 #10524)
- B. D. Bojanov, M. Cvetanov, An extremal problem, (Russian) Annuaire Univ. Sofia, Fac. Math. 67 (1972/73), 195–203 (1976). MR0425423 (54 #13378)
- B. D. Bojanov, An estimate for the ε-entropy of the space of functions analytic in the unit disc, (Bulgarian) Annuaire Univ. Sofia, Fac. Math. 67 (1972/73), 191–194 (1976). MR0430628 (55 #3633)
- B. D. Bojanov, Optimal speed of integration and ε-entropy of a certain class of analytic functions, (Russian) Mat. Zametki 14 (1973), 3–10. MR0342930 (49 #7674)
- B. D. Bojanov, The approximation of functions by positive linear operators, (Bulgarian) Bulgar. Akad. Nauk. Izv. Mat. Inst. 14 (1973), 179–187. MR0350277 (50 #2770)
- B. D. Bojanov, The optimal method for the integration of a certain class of analytic functions, (Russian) in Mathematics and Mathematical Education (Proc. Second Spring Conf. Bulgarian Math. Soc., Vidin, 1973), pp. 75–86. Izdat. Blgar. Akad. Nauk., Sofia, 1974. MR0352822 (50 #5308)

- B. D. Bojanov, Best quadrature formula for a certain class of analytic functions, Zastos. Mat. 14 (1974), 441–447. MR0362846 (50 #15284)
- B. D. Bojanov, On an optimal quadrature formula, C. R. Acad. Bulgare Sci. 27 (1974), 619–621. MR0365998 (51 #2250)
- 17. B. D. Bojanov, Optimal methods of interpolation in $W^{(r)}L_q(M; a, b)$, C. R. Acad. Bulgare Sci. **27** (1974), 885–888. MR0382910 (52 #3792)
- B. D. Bojanov, New best quadrature formulas, C. R. Acad. Bulgare Sci. 27 (1974), 1625–1627. MR0362847 (50 #15285)
- 19. B. D. Bojanov, New best quadrature formulae, Serdica 1 (1975), 110–120. MR0390609 (52 #11434)
- B. D. Bojanov, Best interpolation methods for certain classes of differentiable functions, (Russian) Mat. Zametki 17 (1975), 511–524. MR0393937 (52 #14744)
- 21. B. D. Bojanov, A note on the $W^{(n)}_{\infty}$ extremal problem of Favard, C. R. Acad. Bulgare Sci. 28 (1975), 1163–1166. MR0393938 (52 #14745)
- B. D. Bojanov, Best cubature formulas, Serdica 2 (1976), 42–52. MR0412697 (54 #819)
- B. D. Bojanov, Best methods of integration with a weight for classes of differentiable functions, (Russian) *Eesti NSV Tead. Akad. Toimetised Füüs.-Mat.* 25 (1976), 209–212. MR0415162 (54 #3253)
- B. D. Bojanov, Optimal methods of integration in the class of differentiable functions, Zastos. Mat. 15 (1976), 105–115. MR0417648 (54 #5698)
- B. D. Bojanov, V. M. Veselinov, A general method generating estimates for the best Hausdorff approximation, C. R. Acad. Bulgare Sci. 29 (1976), 959–961. MR0433093 (55 #6072)
- B. D. Bojanov, Numerical integration of functions with singularities, (Bulgarian) Mathematics and Mathematical Education (Proc. Third Spring Conf. Bulg. Math. Soc., Burgas, 1974) (Bulgarian), pp. 120–123, Bulgar. Akad. Nauk, Sofia, 1976. MR0574603
- B. D. Bojanov, Best reconstruction of differentiable periodic functions from their Fourier coefficients, (Russian) Serdica 2 (1976), 300–304. MR0510856 (58 #23311)
- B. D. Bojanov, Characterization and existence of optimal quadrature formulas for a certain class of differentiable functions, (Russian) Dokl. Akad. Nauk SSSR 232 (1977), 1233–1236. MR0442556 (56 #937)
- B. D. Bojanov, V. G. Chernogorov, An optimal interpolation formula, J. Approx. Theory 20 (1977), 264–274. MR0447878 (56 #6188)
- 30. B. D. Bojanov, Best approximation of linear functionals in W_p^r , Pliska Studia Math. Bulgar. 1 (1977), 100–111. MR0473657 (57 #13322)
- B. D. Bojanov, Existence of extended monosplines of least deviation, Serdica 3 (1977), 261–272.
- B. D. Bojanov, Existence of optimal quadrature formulae with preassigned multiplicities of nodes, C. R. Acad. Bulgare Sci. 30 (1977), 639–642. MR0493096 (58 #12133)
- B. D. Bojanov, A note on the optimal approximation of smooth periodic functions, C. R. Acad. Bulgare Sci. 30 (1977), 809–812. MR0481756 (58 #1855)

- B. D. Bojanov, Existence of extended monosplines of least deviation, C. R. Acad. Bulgare Sci. 30 (1977), 985–988.
- B. D. Bojanov, Extremal problems in a set of polynomials with fixed multiplicities of zeros, C. R. Acad. Bulgare Sci. 31 (1978), 377–380.
- B. D. Bojanov, Existence and characterization of optimal quadrature formulas for a certain class of differentiable functions, J. Approx. Theory 22 (1978), 262–283. MR0467121 (57 #6988)
- B. D. Bojanov, The existence of optimal quadrature formulae with prescribed multiplicities of the nodes, (Russian) Mat. Sb. (N.S.) 105(147) (1978), 342– 370, 463. MR0483318 (58 #3331)
- B. D. Bojanov, Favard's interpolation problem for periodic functions, in *Fourier Analysis and Approximation Theory* (Proc. Colloq., Budapest, 1976), Vol. I, pp. 161–173, Colloq. Math. Soc. János Bolyai, **19**, North-Holland, Amsterdam-New York, 1978. MR0540297 (81h:42007)
- B. D. Bojanov, A generalization of Chebyshev polynomials, J. Approx. Theory 26 (1979), 293–300. MR0550677 (81a:41011)
- V. M. Tihomirov, B. D. Bojanov, Some convex problems of approximation theory, (Russian) Serdica 5 (1979), 83–96. MR0554465 (81a:41062)
- B. D. Bojanov, Uniqueness of optimal quadrature formulae, (Russian) Dokl. Akad. Nauk SSSR 248 (1979), 272–274. MR0553186 (81b:41075)
- B. D. Bojanov, On the existence of optimal quadrature formulae for smooth functions, *Calcolo* 16 (1979), 61–70. MR0555454 (81c:41067)
- B. D. Bojanov, Uniqueness of the monosplines of least deviation, in *Numerische Integration*, (G. Hämmerlin, Ed.), pp. 67–97, ISNM, 45, Birkhäuser, Basel, 1979. MR0561282 (81f:65009)
- B. D. Bojanov, Uniqueness of optimal nodes of quadrature formulas, (Russian) C. R. Acad. Bulgare Sci. 33 (1980), 167–169. MR0574452 (81e:41043)
- B. D. Bojanov, Perfect splines of least uniform deviation, Anal. Math. 6 (1980), 185–197. MR0592094 (82d:41009)
- 46. B. D. Bojanov, Existence and characterization of monosplines of least L_p deviation, in *Proc. Intl. Conf. Const. Functions Theory*, Blagoevgrad 1977, pp. 249–268, Sofia, 1980.
- B. D. Bojanov, Uniqueness of the optimal nodes of quadrature formulae, Math. Comp. 36 (1981), 525–546. MR0606511 (82d:65024)
- B. D. Bojanov, An extension of Markov's inequality, (Russian) Dokl. Akad. Nauk SSSR 262 (1982), 13–15.
- B. D. Bojanov, Proof of a conjecture of Erdös about the longest polynomial, Proc. Amer. Math. Soc. 84 (1982), 99–103. MR0633287 (83e:41018)
- B. D. Bojanov, An extension of the Markov inequality, J. Approx. Theory 35 (1982), 181–190. MR0662166 (83h:41013)
- B. D. Bojanov, Oscillating polynomials of least L₁-norm, in Numerical Integration, (G. Hämmerlin, Ed.), pp. 25–33, ISNM 57, Birkhäuser Verlag, Basel, 1982.
- B. D. Bojanov, L₁-approximation by Hermite interpolating polynomials with free nodes, C. R. Acad. Bulgare Sci. 35 (1982), 1049–1051. MR0687401 (84d:41008)

- B. D. Bojanov, An extremal problem for polynomials, in *Constructive Function Theory '81*, pp. 229–233, Sofia, 1983.
- B. D. Bojanov, A generalization of Chebyshev polynomials, II. *Pliska Stud. Math. Bulgar.* 5 (1983), 93–96. MR0704139 (85b:41003)
- B. D. Bojanov, Addendum to: "Proof of a conjecture of Erdös about the longest polynomial" [*Proc. Amer. Math. Soc.* 84 (1982), 99–103; MR 83e:41018]. *Proc. Amer. Math. Soc.* 89 (1983), 188. MR0706541 (84m:41021)
- J.-E. Andersson, B. D. Bojanov, A note on the optimal quadrature in H^p, Numer. Math. 44 (1984), 301–308. MR0753961 (86a:65023)
- B. D. Bojanov, Sendov's conjecture on critical points of polynomials, (Bulgarian) Fiz.-Mat. Spis. Bulgar. Akad. Nauk. 26 (1984), 140–150. MR0769951 (86c:30010)
- B. D. Bojanov, Q. I. Rahman, J. Szynal, On a conjecture of Sendov about the critical points of a polynomial, *Math. Z.* **190** (1985), 281–285. MR0797543 (86j:30010)
- B. D. Bojanov, Q. I. Rahman, J. Szynal, On a conjecture about the critical points of a polynomial, in *Delay Equations, Approximation and Application (Mannheim,* 1984), 83–93, ISNM, 74, Birkhäuser, Basel, 1985. MR0899090 (88e:30013)
- B. D. Bojanov, Comparison theorems in optimal recovery, in *Optimal Algorithms*, (B. Sendov, Ed.), pp. 15–50, Sofia, BAN, 1986.
- B. D. Bojanov, D. Braess, N. Dyn, Generalized Gaussian quadrature formulas, J. Approx. Theory 48 (1986), 335–353. MR0865436 (88c:41049)
- B. D. Bojanov, L-approximations with constraints, (Russian) in Proc. Intl. Conf. on the Approximation of Functions, pp. 69–71, Kiev, 1983, Nauka, Moscow, 1987.
- B. D. Bojanov, D. R. Huang, Periodic monosplines and perfect splines of least norm, Constr. Approx. 3 (1987), 363–375. MR0904341 (88j:41029)
- B. D. Bojanov, σ-Perfect splines and their application to optimal recovery problems, J. Complexity 3 (1987), 429–450. MR0919098 (89m:65018)
- B. D. Bojanov, B-splines with Birkhoff knots, C. R. Acad. Bulgare Sci. 40 (1987), 11–14. MR0920902 (89d:41020)
- B. D. Bojanov, B-splines with Birkhoff knots, Constr. Approx. 4 (1988), 147– 156. MR0932651 (89c:41001)
- 67. B. D. Bojanov, G. R. Grozev, A note on the optimal recovery of functions in H^{∞} , J. Approx. Theory **53** (1988), 67–77. MR0937144 (89g:41012)
- B. D. Bojanov, D. R. Huang, Comparison of recovery schemes based on end-point values, *Math. Balkanica (N.S.)* 2 (1988), 11–17. MR0965896 (90b:41029)
- 69. B. D. Bojanov, D. R. Huang, On the optimal quadrature formulas in W_q^r of quasi-Hermitian type, Approx. Theory Appl. 4 (1988), 13–22. MR0986337 (90g:41042)
- 70. R. B. Barrar, B. D. Bojanov, H. L. Loeb, Generalized polynomials of minimal norm, J. Approx. Theory 56 (1989), 91–100. MR0977876 (90b:41047)
- B. D. Bojanov, G. Nikolov, Comparison of Birkhoff type quadrature formulae, Math. Comp. 54 (1990), 627–648. MR1010595 (91e:65034)
- 72. B. D. Bojanov, D. R. Huang, Comparison of optimal quadrature formulas, Numer. Math. 56 (1990), 817–825. MR1035180 (91d:65041)

- B. D. Bojanov, Optimal reconstruction of differentiable functions, (Russian) Mat. Sb. 181 (1990), 334–353; translation in Math. USSR-Sb. 69 (1991), 357– 377. MR1049993 (91c:41066)
- B. D. Bojanov, On the total positivity of the truncated power kernel, *Colloq. Math.* **60/61** (1990), 593–600. MR1096399 (92b:41004)
- B. D. Bojanov, G. R. Grozev, A. A. Zhensykbaev, Generalized Gaussian quadrature formulas for weak Tchebyshev systems, in *Optimal Recovery*, (B. Bojanov and H. Wozniakowski, Eds.), pp. 115–140, Nova Sci. Publ., Commack, NY, 1992. MR1219542 (94h:41062)
- B. D. Bojanov, Elementary proof of the Remez inequality, Amer. Math. Monthly 100 (1993), 483–485. MR1215537
- B. D. Bojanov, Interpolation by periodic splines with Birkhoff knots, Numer. Math. 65 (1993), 63–75. MR1217439 (94e:41022)
- B. D. Bojanov, Polynomial inequalities, in *Open Problems in Approximation Theory*, (B. Bojanov, Ed.), pp. 25–42, CST Publishing, Singapore, 1994.
- B. D. Bojanov, Characterization of the smoothest interpolant, SIAM J. Math. Anal. 25 (1994), 1642–1655. MR1302167 (95h:41002)
- B. D. Bojanov, Optimal recovery of functions and integrals, in *Proc. First European Congress of Mathematics*, Vol. I (Paris, 1992), 371–390, Progr. Math., **119**, Birkhäuser, Basel, 1994. MR1341829 (96e:41032)
- B. D. Bojanov, Q. I. Rahman, On certain extremal problems for polynomials, J. Math. Anal. Appl. 189 (1995), 781–800. MR1312553 (95k:41010)
- B. D. Bojanov, An inequality of Duffin and Schaeffer type, *East J. Approx.* 1 (1995), 37–46. MR1404340 (97g:41016)
- B. D. Bojanov, On a formula of Obreshkoff. Session Dedicated to the Centenary of the Birth of Nikola Obreshkoff (Sofia, 1996). Annuaire Univ. Sofia, Fac. Math. Inform. 89 (1995), 19–21 (1998). MR1757861
- B. D. Bojanov, A. K. Varma, On a polynomial inequality of Kolmogoroff's type, *Proc. Amer. Math. Soc.* **124** (1996), 491–496. MR1291763 (96i:41010)
- B. D. Bojanov, G. Nikolov, Duffin and Schaeffer type inequality for ultraspherical polynomials, J. Approx. Theory 84 (1996), 129–138. MR1370594 (96k:41013)
- B. D. Bojanov, Turán's inequalities for trigonometric polynomials, J. London Math. Soc. (2) 53 (1996), 539–550. MR1396717 (98e:42002)
- B. D. Bojanov, On a quadrature formula of Micchelli and Rivlin, J. Comput. Appl. Math. **70** (1996), 349–356. MR1399878 (97d:41031)
- A. Andreev, V. I. Berdyshev, B. D. Bojanov, B. S. Kashin, S. V. Konyagin, S. M. Nikolskii, K. I. Oskolkov, P. Petrushev, B. Sendov, S. A. Telyakovskii, V. N. Temlyakov, In memory of Sergei Borisovich Stechkin [1920–1995], *East J. Approx.* 2 (1996), 131–133. MR1407059
- B. D. Bojanov, Total positivity of the spline kernel and its applications, in *Total Positivity and its Applications*, (M. Gasca and C. A. Micchelli, Eds.), pp. 3–34, Math. Appl., **359**, Kluwer Acad. Publ., Dordrecht, 1996. MR1421594 (98a:41014)
- 90. B. D. Bojanov, A Jackson type theorem for Tchebycheff systems, *Math. Balkanica* (N.S.) **10** (1996), 73–82. MR1429151 (97k:41007)

- B. D. Bojanov, Gaussian quadrature formulae for Tchebycheff systems, *East J. Approx.* 3 (1997), 71–88. MR1446172 (98c:41043)
- B. D. Bojanov, G. Petrova, On minimal cubature formulae for product weight functions, J. Comput. Appl. Math. 85 (1997), 113–121. MR1482159 (98m:65032)
- B. D. Bojanov, A. Guessab, Gaussian quadrature formula of Birkhoff's type, Calcolo 34 (1997), 41–50 (1998). MR1658103 (99j:65033)
- 94. B. D. Bojanov, Remarks on the Jackson and Whitney constants, in *Recent Progress in Inequalities*, (G. Milanović, Ed.), pp. 161–174, Math. Appl., 430, Kluwer Acad. Publ., Dordrecht, 1998. MR1609935 (99c:41034)
- B. D. Bojanov, Notes on miscellaneous approximation problems, in *Approximation Theory*, 97–113, Monogr. Textbooks Pure Appl. Math., 212, Dekker, New York, 1998. MR1625222 (99d:41001)
- 96. B. D. Bojanov, G. Petrova, Numerical integration over a disc. A new Gaussian quadrature formula, *Numer. Math.* 80 (1998), 39–59. MR1642507 (99j:41048)
- 97. B. D. Bojanov, A note on the Hobby-Rice and Gauss-Krein theorems, East J. Approx. 4 (1998), 371–377. MR1656138 (99i:41027)
- B. D. Bojanov, Markov interlacing property for perfect splines, J. Approx. Theory 100 (1999), 183–201. MR1710559 (2000f:41010)
- B. D. Bojanov, A. Sri Ranga, Some examples of moment preserving approximation, in *Continued Fractions: from Analytic Number Theory to Constructive Approximation*, (B. Bernt and F. Gesztesy, Eds.), pp. 57–70, Contemp. Math., 236, Amer. Math. Soc., Providence, RI, 1999. MR1665362 (2000m:41041)
- B. D. Bojanov, N. Naidenov, An extension of the Landau-Kolmogorov inequality. Solution of a problem of Erdös, J. d'Anal. Math. 78 (1999), 263–280. MR1714429 (2000h:41014)
- 101. B. D. Bojanov, L. Gori, Moment preserving approximations. Math. Balkanica (N.S.) 13 (1999), 385–398. MR1779971 (2002a:41028)
- 102. B. D. Bojanov, D. P. Dryanov, W. Haussmann, G. P. Nikolov, Best one-sided L¹-approximation by blending functions, in Advances in Multivariate Approximation, (W. Haussmann, K. Jetter and M. Reimer, Eds.), pp. 85–106, Math. Res., 107, Wiley-VCH, Berlin, 1999. MR1797224 (2001j:41037)
- 103. B. D. Bojanov, G. Petrova, Uniqueness of the Gaussian quadrature for a ball, J. Approx. Theory 104 (2000), 21–44. MR1753510 (2001d:41030)
- 104. B. D. Bojanov, D. K. Dimitrov, Gaussian extended cubature formulae for polyharmonic functions, *Math. Comp.* **70** (2001), 671–683. MR1697644 (2001g:41046)
- 105. B. D. Bojanov, P. P. Petrov, Gaussian interval quadrature formula, *Numer. Math.* 87 (2001), 625–643. MR1815728 (2001k:65043)
- 106. B. D. Bojanov, W. Haussmann, G. P. Nikolov, Bivariate polynomials of least deviation from zero, *Canad. J. Math.* **53** (2001), 489–505. MR1827818 (2002b : 41004)
- 107. B. D. Bojanov, Cubature formulae for polyharmonic functions, in *Recent Progress in Multivariate Approximation*, (W. Haussmann, K. Jetter and M. Reimer, Eds.), pp. 49–74, ISNM, **137**, Birkhäuser, Basel, 2001. MR1877497 (2003a:41029)
- 108. B. D. Bojanov, Y. Xu, On a Hermite interpolation by polynomials of two variables, SIAM J. Numer. Anal. 39 (2001/02), 1780–1793. MR1885716 (2002m:41002)

- 109. B. D. Bojanov, An extension of the Pizzetti formula for polyharmonic functions, Acta Math. Hungar. 91 (2001), 99–113. MR1912362 (2003e:31004)
- B. D. Bojanov, N. Naidenov, Exact Markov-type inequalities for oscillating perfect splines, *Constr. Approx.* 18 (2002), 37–59. MR1866379 (2002h:41013)
- 111. B. D. Bojanov, Notes on miscellaneous problems, East J. Approx. 8 (2002), 123–130. MR1916329 (2003d:41001)
- B. D. Bojanov, N. Naidenov, Examples of Landau-Kolmogorov inequality in integral norms on a finite interval, J. Approx. Theory 117 (2002), 55–73. MR1920119 (2003d:41021)
- B. D. Bojanov, Markov-type inequalities for polynomials and splines, *Approximation Theory X*, (C. K. Chui, L. L. Schumaker and J. Stoeckler, Eds.), 31–90, Innov. Appl. Math., Vanderbilt Univ. Press, Nashville, 2002. MR1924851 (2003f:41018)
- 114. B. D. Bojanov, Blagovest Sendov and his contribution to approximation theory, Approximation Theory, ii–xviii, DARBA, Sofia, 2002. MR1935800 (2003m:01043)
- 115. B. D. Bojanov, Y. Xu, On polynomial interpolation of two variables, J. Approx. Theory **120** (2003), 267–282. MR1959868 (2003m:41001)
- B. D. Bojanov, P. Petrov, Uniqueness of the Gaussian interval quadrature formula, Numer. Math. 95 (2003), 53–62. MR1993938 (2004g:65020)
- 117. B. D. Bojanov, I. K. Georgieva, Interpolation by bivariate polynomials based on Radon projections, *Studia Math.* **162** (2004), 141–160. MR2046566 (2005f:41003)
- B. D. Bojanov, N. Naidenov, Alternation property and Markov's inequality for Tchebycheff systems, *East J. Approx.* **10** (2004), 481–503. MR2101505 (2005h:41019)
- 119. B. D. Bojanov, Majorization of polynomials on the plane, *Indag. Math. (N.S.)* 15 (2004), 469–483. MR2114931 (2005k:41037)
- 120. B. D. Bojanov, V. Gushev, A quadrature formula of Gaussian type on the sphere, East J. Approx. 11 (2005), 119–130. MR2151611 (2006b:41039)
- 121. B. D. Bojanov, Y. Xu, Reconstruction of a polynomial from its Radon projections, SIAM J. Math. Anal. 37 (2005), 238–250. MR2176931 (2006f:42005). Erratum, SIAM J. Numer. Anal. 45 (2007), 1799–1800. MR2338410 (2008f:41009)
- B. D. Bojanov, P. Petrov, Gaussian interval quadrature formulae for Tchebycheff systems, SIAM J. Numer. Anal. 43 (2005), 787–795. MR2177891 (2006h:65030)
- 123. B. D. Bojanov, Quadrature formulae of non-standard type, *Rend. Mat. Appl.* 25 (2005), 195–221. MR2197879 (2007d:41031)
- 124. B. D. Bojanov, Optimal quadrature formulas, (Russian) Uspekhi Mat. Nauk 60 (2005), 33–52; translation in Russian Math. Surveys 60 (2005), 1035–1055. MR2215753 (2006m:41047)
- 125. B. D. Bojanov, N. Naidenov, Majorization of polynomials on the plane. II, *East J. Approx.* **12** (2006), 189–202. MR2242667 (2007b:41018)
- 126. B. D. Bojanov, Interpolation and integration based on averaged values, in Approximation and Probability, (T. Figiel and A. Kamont, Eds.), pp. 25–47, Banach Center Publ., 72, Polish Acad. Sci., Warsaw, 2006.
- 127. B. D. Bojanov, Lubomir Tschakaloff (1886–1963), Serdica Math. J. 33 (2007), iii–iv. MR2313791

- 128. B. D. Bojanov, Nikola Obreshkoff (1896–1963), Serdica Math. J. 33 (2007), v–vi. MR2313792
- B. D. Bojanov, 50 years of Sendov's conjecture, (Bulgarian) J. Bulg. Acad. Sciences 6 (2007), 27–33.
- 130. B. D. Bojanov, G. Petrova, Quadrature formulae for Fourier coefficients, J. Comp. Applied Math., to appear.
- 131. B. D. Bojanov, Extremal problems for polynomials in the complex plane, in *Proc. Int. Conf. "Approximation and Computation"*, Nis, 2008, to appear.
- 132. B. D. Bojanov, N. Naidenov, On oscillating polynomials, submitted for publication.

Books

- B1. B. D. Bojanov, Methods for Approximate Evaluation of Integrals (in Bulgarian), Nauka i Izkustvo, Sofia 1978. MR0518460 (80b:41008)
- B2. B. D. Bojanov, Interpolation (in Bulgarian), Narodna Prosveta, Sofia 1984.
- B3. B. D. Bojanov, H. Hakopian, *Theory of Spline Functions* (in Bulgarian), Nauka i Izkustvo, Sofia 1990.
- B4. B. D. Bojanov, H. Hakopian, A. Sahakian, Spline Functions and Multivariate Interpolations, Mathematics and its Applications, 248, Kluwer Acad. Publishers Group, Dordrecht, 1993. MR1244800 (94k:41001)
- B5. B. D. Bojanov, *Lectures on Numerical Analysis* (in Bulgarian), DARBA, Sofia 1995.

Books Edited

- E1. B. D. Bojanov, H. Wozniakowski, *Optimal Recovery*, Nova Science Publishers, New York, 1992.
- E2. B. D. Bojanov, *Open Problems in Approximation Theory*, CST Publishing, Singapore, 1994.
- E3. B. D. Bojanov, Approximation Theory: A volume dedicated to Blagovest Sendov, DARBA, Sofia, 2002.
- E4. B. D. Bojanov, *Constructive Theory of Functions: Varna 2002*, DARBA, Sofia, 2003.
- E5. B. D. Bojanov, Constructive Theory of Functions: Varna 2005, Marin Drinov Academic Publishing House, Sofia, 2006.