

In Memoriam

In Memoriam: Ranko Bojanic
November 12, 1924–February 21, 2017



Ranko Bojanic on July 13 of 1989 in Columbus, Ohio, at PaulN's house.

Ranko Bojanic (aka Bojanić) was born on November 12, 1924 in Breza, Yugoslavia (currently Bosnia and Hercegovina) and he was raised in its capital, Belgrade. Ranko studied mathematics at the University of Belgrade where he was awarded a bachelor's degree in 1950. After graduating, he continued his studies at the Mathematics Institute of the Serbian Academy of Sciences, where he was awarded Doctorate in Mathematics in 1953. Jovan Karamata was his doctoral advisor. Permanent positions were scarce in the 1950s Europe so he accepted several two year visiting positions first at various universities in Yugoslavia and then at the Tata Institute of Fundamental Research in Bombay, India. While there, Gábor Szegő invited him for a one year visiting position at Stanford University, arriving in the United States in 1959. From Stanford he moved to the University of Notre Dame. When his chairman, Arnold Ross moved from Notre Dame to Ohio State University, Ranko came with him and joined the Department of Mathematics at Ohio State as Associate Professor in 1963. He was promoted to Professor in 1966 and remained there until his retirement in 1995.

Ranko's true passions were research and teaching, with his research focus in classical approximation theory. He was full of ideas, which he liked to share. In the 1970s and 1980s, his office often became a gathering place for students, colleagues, and visitors alike where exciting mathematical ideas in analysis were discussed, and problems were solved. These meetings sometimes took place with supplies of sweets that Ranko often kept around for visitors.

His interactions with other mathematicians, whether at Ohio State, or at various international conferences, often resulted in collaborative work.

After his retirement in 1995, Ranko continued to be professionally active. He still taught for a few more years; he continued his work as an Associate Editor of the Journal of Approximation Theory all the way through 2016; he also continued his work as the Managing Editor of the Journal of Number Theory through 2010. His last paper, in collaboration with Professor Eugene Seneta, will be published post mortem. After a long, courageous, partially philosophical, and partially optimistic fight with various ailments, including multiple occurrences of cancer, Ranko died at his home on February 21 or 2017 in Columbus in the presence of his beloved wife Olga, and children Mira and Ivan.



Ranko & Olga Bojanic in September of 1989 in Maratea, Italy.

Reminiscences by colleagues, friends, and students



Ron DeVore in May of 1985 in Hangzhou, China.

Ronald A. DeVore. I came to do my graduate studies at the Ohio State University, located on the banks of the Olentangy river, in the Fall of 1964. The first thing I learned was how to pronounce “Olentangy” (soft “g”). The second was that the library was closed on football Saturdays. Football was king and epitomized by the story of Woody Hayes (the famous football coach to you less learned) coming home from late night football preparations, crawling into bed with his wife shouting “*God, your feet are cold*”, to which he replied “*you can call me Woody when we are alone together*”.

While I spent some time watching the sports at OSU via discounted tickets for students, I was there to learn some mathematics. At that time, all mathematics graduate students were required

to take the first graduate course in Analysis (measure theory +) and the first graduate course in Algebra (van der Waerden). There was also the requirement that a third course be chosen of the student's choice. For whatever reason, probably pure chance, I chose a course on Approximation Theory taught by Professor Ranko Bojanic. As the course proceeded, it seemed to me that the material in the course (interpolation of data, polynomial approximation, etc.) could easily be justified as having some relevance to real world problems. In any case, I felt quite comfortable.

The course itself was meticulously presented. It was clear it had been developed over the years with polished notes and an array of problems to round out and deepen the material. Professor Bojanic himself was matter of fact in his presentation, rather low key with an enticing Serbian accent. Coming from a Polish neighborhood in Detroit I was very much at home.

One nice thing about Approximation Theory is that it is very accessible and easy to immediately begin posing research questions. This was the case with me and I began to prove little tidbits on polynomial approximation. Near the end of my first quarter of study I mentioned to Professor Bojanic that I had proved some results about one-sided approximation. He said he would like to hear them and we settled on a meeting time. We were joined by Professor Bogdan Baishanski in a small seminar room. Bogdan and Ranko were good friends and although Bogdan had some interest in Approximation Theory, his main interests, at least at that time, were in Harmonic Analysis. I later learned a lot from courses I took from him on that subject. Anyway, I began to tell them what I could prove and was met with phrases like *"you should stop now and not embarrass yourself since what you propose to prove is obviously not true"*. Nevertheless, I continued my presentation, met with the same refrain at many junctures. After a somewhat exhausting experience, I reached the end of what I wanted to prove, only to be told that the whole matter was quite trivial and now I should be embarrassed by its triviality. Later I learned that the whole experience was *modus operandi* when discussing mathematics with many Eastern Europeans.



Bogdan & Jacqueline Baishanski in June of 1987 at PaulN's first retirement party.

After taking three one quarter courses in Approximation Theory, I knew quite a bit about the subject and would say I was a full blown researcher. I do not ever recall formally asking Ranko to be my advisor, it just seemed to happen since we talked daily about problems and began working together. He became much more positive in his attitude about where I was going and offered much fatherly advice. One thing I use till this day is that when you engage something complicated, think of it in its simplest cases, before attacking its full blown version. Also, organize your thoughts from coarse to fine and move on to the coarse level before taking the time to check the fine points.

Ohio State mathematics was very active at that time with a constant stream of distinguished visitors. I remember Calderón, Zygmund, Pólya, and Turán, all taking time out during their visits

to talk with me and to say they heard very good things about me. I am sure this was all due to pre-advertisement by Ranko and Bogdan.

There was also the social scene with parties and politics. It seemed Ranko and Bogdan did not always see eye to eye in the political arena but I never understood the subtle differences they so strongly held onto. It was good to see them in heated arguments but remain good friends. Ranko was also known as one who could hold his liquor. When putting on such a demonstration, he would become quiet but generate a very sly grin on his face. I have no idea why he was smiling and wish I knew.

I remember one small conference where Ranko asked me to go for a drink with him in the evening. After two cognacs, I was already wasted and told him I needed to bow out. He insisted on staying and paying the bill which I later found out was more than a hundred dollars - a lot of money in the 1960s and drinks were not that expensive. By the way, the mathematical culture seemed different in those days: coffee in coffee shops, serious drinking, writing on table clothes, lots of arguments with harsh words, but always forgotten the next day. I enjoyed the scene with a lot of teasing and directness. This was the way for a young boy growing up in a Polish neighborhood of Detroit.

Ranko and Bogdan remained my friends through the years. They were good people, in and out of mathematics. I was fortunate they came to my 60th with their wives. It was good to see that they had not changed much. I was fortunate that chance brought me to Ranko. He was a major part of my life and I will miss him dearly.



Zita Divis on July 13 of 1989 in Columbus, Ohio, at PaulN's house.

Zita M. Divis. Ranko and his wife, Olga Bojanic, were also personal friends so I knew him somewhat better than most. He had many interests besides mathematics, and somehow managed to find time to keep up with them. He was an avid shortwave radio builder (and listener). In fact, Olga used to complain that he kept the basement cluttered with all his electronics. But that helped him to stay very well informed about politics in the world, particularly Europe. Discussions about politics in his office could become quite heated at times.

Ranko's interests, however, were much broader. He turned out to be a real connoisseur of European cinema. At the time I joined the department, there was a very active Film Club on campus, presenting many outstanding movies from all over the world. Ranko and Olga (and Bogdan with his wife) would often join us there. We had many lively discussions afterwards. Ranko's movie collection was quite extensive. Later in life, as movies became accessible on the computer, he simply used the modern technology to find what was available.

Ranko was very generous with his time not only with regard to mathematics, as evidenced in many professional meetings taking place in his office where he freely shared his ideas (these are described by others) but also in other situations. I will mention one such example. When my young son became interested in toy trains, Ranko taught him about electrical circuits and also

how to solder. The result was an amazing railroad display in our basement. I am among many who will sorely miss him.



Hrushikesh N. Mhaskar in May of 1988 on the top of Masada in Israel with Jana DeVore in the background.

Hrushikesh N. Mhaskar. I met Professor Bojanic first when I enrolled at Ohio State as a graduate student in 1976. He was a very kind and unselfish man.

At the time when my Ph.D. advisor, Professor Géza Freud, died suddenly (September 27, 1979), I had not written my thesis yet, but done much more research than what was eventually included in it, leaving the rest for publications (about 6 or 7) after getting my Ph.D. degree. However, it was necessary to obtain my M.S. degree in Computer Science (CS) before completing the formalities for the Ph.D. In turn, this required me to enroll in the CS department. Since Ranko had a formal position there, I thought I might as well choose him as an advisor in the CS department and then finish the Ph.D. formalities with Ranko as the formal advisor when I returned to the Department of Mathematics. Ranko advised me not to do so but to choose Paul Nevai instead.¹ This turned out to be a good decision. Paul gave me some very practical advice on some important matters, not limited just to the content of my thesis; indeed continues to do so even now when needed.

Around 1985, I sent him a manuscript of my paper on convergence of Jacobi expansions at points of discontinuity in high order derivatives of a function. It turned out that he was working on the same problem but had some technical difficulties. So, I proposed to do it as a joint paper. Again, he thought it would be in my best interest to publish the paper as a sole author, although he read the draft carefully and made copious suggestions. He then offered that I coauthor with him the Hermite polynomial version, combining some asymptotic results which he had obtained, with the more general estimates which I had obtained for Freud polynomials as they applied to the Hermite case. He suggested I should do the more general Freud polynomial version myself, which he then later accepted for JAT. He told me that we should rewrite our joint paper several times, so that I will learn how to write well! This joint paper will probably never materialize now.

On the lighter side, he gave me a (free) ride in 1978 for a meeting in Rhode Island, where Donald J. Newman gave his 10 lectures on “*Approximation with Rational Functions*”. Throughout the entire 15 h or so of the journey, we spoke hardly 5 sentences to each other, perhaps, not knowing what to say. On the return trip, Newman was with us until we came to New Jersey, and they were talking constantly with each other about people and subjects I did not know anything about!

I wish Ranko a pleasant onward journey in whatever re-incarnations he still needs to take, if any.

¹ PaulN: I was unaware of this up until I read about it here.



Ranko Bojanic and Paul N. Nevai on July 13 of 1989 in Columbus, Ohio, at Paul N.'s house.

Paul Nevai. I met Ranko for the first time in September of 1973 at a conference in Cluj, Romania (the birthplace of my paternal grandmother when it was in the Austro-Hungarian Empire and where, upon arrival, the windshield wipers and the side-mirrors of my Zhiguli were instantaneously stolen). He was one of the first Yankee mathematicians I ever saw, the other one being Arun Varma, and I still vividly remember him with a fancy camera hanging from his neck — how I envied him. At that time my English was not yet ready to carry a conversation so that it was a special pleasure for me to be able to communicate with him in Russian. Ranko was then still in his forties but looked exactly the same way as he did when he was 90 years old.

Our second encounter was in August of 1974, just days after Nixon's resignation, in Madison, Wisconsin, where we both participated in the first joint US–Hungarian approximation theory meeting. By then my English improved a bit to the extent that I was able to detect that Ranko's English was not that great either. Later I found out that Ranko spoke mostly Serbian both at home and at work where he primarily communicated with his best friend, the distinguished approximator/analyst Bogdan Baishanski.

Ranko was happy to talk Russian and I was happy to talk to someone who actually understood my English so we ended up spending a lot of time together. As it happened, Ranko, Ron DeVore, and I ended up sitting together at the conference dinner, and Ron was having fun trying to figure out how it felt to live in a country where people were not “free” so to speak.² Not being able to speak my mind freely, I felt kind of uncomfortable but by the middle of the dinner I decided that I am going to defect and will live in the US so that, among others, I would no longer be the subject of such a ridicule. Of course, then it was only a plan but in less than 3 months it became a reality.

I guess I must have made a positive impression on Ranko because 2 years later he, together with Géza Freud, was instrumental in arranging for a job for me at the³ Ohio State University where I was hired without a personal interview (because I was stuck in Paris with my immigration process slowly inching forward).

While I was a PostDoc in Madison, Wisconsin, I came to Columbus for a short visit in October of 1976 to inspect the place where I was going to start my professorial career the next year. I will never forget the “welcoming committee” of Ranko, Bogdan, and Géza, and the huge smiles on their faces. I immediately knew I will love being their colleague although I did not imagine that I will actually spend most of my life in Columbus and will retire from OSU after 35+ years of service.

So this is how it started.

² Ron, do you still remember this?

³ Then the article “the” was not capitalized yet.

When I started at Ohio State in 1977, Ranko spent much of his time dealing with graduate students and departmental affairs until the time when the first programmable calculators (HP) and then personal computers (Radio Shack and such) started to become available. Ranko immediately sensed that a new era started and when the first Macintosh came in 1984, Ranko's life drastically changed. From then on Ranko dedicated his life to programming the Macintosh (mostly in Basic and Pascal) to solve approximation theoretic problems such as implementations of the Remez algorithm. For some time, we even ran a seminar discussing mathematical issues of computer programming such as making computers understand math papers.

If I had to summarize Ranko's life after 1985, I could do it with just a few words: his son Ivan's dog, the OSU squirrels, and his beloved Macintosh.

I could write an essay about these items individually but I will spare the reader from the details. OK, just a few. When Ivan's job took him to Europe, Ranko inherited his dog who every day accompanied Ranko to OSU. Despite dogs being strictly forbidden in campus buildings and seeing how happy Ranko was while the dog trailed him everywhere he went, no one had the heart to mention this to Ranko. After Ivan's dog died, Ranko had another dog who was the love of his (dog) life. Knowing Ranko, of course it did not matter what the rules were, the dogs were his most loyal friends and companions. Well, maybe. Or maybe not. You see, Ranko also was beloved by all squirrels on campus. Every time Ranko showed up for work, he had a handful of peanuts in his hand, and the campus squirrels patiently surrounded him at the department's main entrance waiting for him to hand each of them some of these goodies.

Although I grew up in Hungary, I never really paid attention to alcohol in general and to fruit brandy in particular. It was Ranko's 90th birthday party when I gave a speech and there somehow it was brought up that Ranko owes his long life to his wife Olga and to slivovitz (plum brandy). Naturally, I decided to try slivovitz myself but it turned out that it was not available in Columbus despite being on the Ohio Division of Liquor Control's approved list. I had to call the agency several times until they finally allotted a few cases to the supermarket I frequent, and ever since then I always have a couple of bottles of slivovitz at home. I must admit, and I hope that Ranko is not listening, slivovitz is overrated. On the other hand, I introduced the 90 year old Ranko to Belgian beer. Amazingly, Ranko, who liked beer, never in his life had what I would call "*quality*" beer until I gave him one Chimay Grande Réserve. Ranko was astonished to see how different it was from his regular staple. I also recall Olga being happy to see that her seriously ill husband was still capable to enjoy small pleasures.

One controversial aspect of living the life of a globetrotter mathematician during the cold war created a situation where I disapproved Ranko's happy-go-lucky take on life. Namely, Ranko, despite being a strong anti-anti-Semite, had amiable relationships during the cold war with several Soviet mathematicians who were anything but pro-Semitic. It was Ranko's nature to be friendly with everyone, including the bad guys. This bothered me but I was unable to explain to Ranko that not all people deserve being treated with respect.

Here are a couple of tidbits I want to record for Ranko. One is that he semi-bitterly complained to me that he never expected to live this long and, therefore, he did not plan his old age properly. Instead of taking a life-annuity, he chose one with a fixed period that one day, after the fixed period expired, simply stopped coming. Also, he would not have retired at the age of 70 had he known that he was going to live 20+ years after that. Another tidbit is that despite being financially well off, he wore the same shoes and pants and drove the same economy car year after year. I did not count the years but it must have been over 20. Also, his office was quite messy and disorganized. Before the reader says that this is typical for mathematicians, let me point out that his wife Olga was a classy lady who was always super elegant and their house

was always beautifully furnished and spotless. Olga and Ranko had a deal: Olga runs Ranko's life at home and Ranko is free to do anything he wants to while outside the home. This system worked quite well. Yet another is that Ranko told me that he never liked to use tools of modern life such as email, texting, or skyping. However, this is not exactly true. Ranko was a serious email user already in the early 1990s and he gave up his habit only when he got very old. Then he forgot what he used to do. In addition, as I learned from Zita Divis, Ranko was actually quite a handyman in his youth and middle-aged years.

One funny story Ranko loved to tell was about a paper he and Bogdan wrote in English that was translated into Russian and published in the Soviet Union in *Mat. Zametki*, and then it ended up being translated back into English when it came out in the *Math. Notes*. However, when I searched MathSciNet, I found no trace of such a paper. There is only one paper listed there that Ranko wrote in Russian (MR0203344 with Tomič) and that was not republished in English. I still like this story because I happen to have two such papers myself, one with Tamás Erdélyi & Michael Ganzburg and the other one with Tamás.

I am very happy that I had the opportunity to hug and kiss Ranko and tell him that I loved him many times. Initially he was a little surprised, perhaps even shocked, but eventually he got used to it and hugged me back.

In case one wonders if, in addition to Olga's extraordinary attachment to her husband, there was any secret explaining Ranko's long life, I can summarize it with the word "stubbornness"; the reader should google "longevity stubborn" and will see that Ranko was the choirboy for the longevity studies.

Finally, let me mention that Ranko was angry, in fact, furious at me once and only once when we represented different sides in a tenure case. Ranko asked me to come to his office and told me privately that no matter what I felt about the case, there is a more important principle at stake, namely, loyalty to one's friends. I thought about it a little while and came to the conclusion that Ranko is right. Life is short and supporting our friends should take precedence over many other issues.

Being religiously non-religious, I cannot really finish by saying "rest my beloved friend in peace" but I can tell you that I will value your friendship and will always miss you while I am still alive.



Eugene Seneta in May of 2017.

Eugene Seneta. The fundamental role played by Karamata's theory of regularly varying functions in probability theory came to be widely recognized by probabilists with the publication in 1966 of Volume 2 of William Feller's "*An Introduction to Probability Theory and Its Applications*" which contained elements of an exposition of the Karamata theory. I needed to

learn this theory for application to certain problems in the theory of simple branching processes, but found Feller's exposition unclear.

I obtained through the University library photocopies of Karamata's early papers, and wrote to Karamata requesting further materials. A package came from Geneva containing among other things, Technical Summary Reports of the Mathematics Research Center, Madison, Wisconsin (1963), at least one by R. Bojanić and J. Karamata, and advice to write for further materials to Karamata's former research students, S. Aljančić, R. Bojanić, and M. Tomić. This may have been just before Karamata's death in Geneva in August 1967.

At the beginning of one of the Technical Summary Reports, there was a clear exposition of the theory of regular variation, written, as I later realized, by Ranko. This was based on two fundamental theorems: the Uniform Convergence Theorem and the Representation Theorem, the bedrock of his exposition of, or application of, regularly varying functions. And there were citations (in a variety of languages all of which I was fortunately able to read) of the fundamental sources in the development of the theory to that point in time. I found the theory thus presented to be remarkably clear and beautiful.

I wrote to Ranko Bojanić at Ohio State and to Miodrag Tomić in Belgrade, and both obliged, Ranko with further Technical Summary Reports, and both with offprints. One of these offprints by Aljančić, Bojanić and Tomić (1956) is now much worn. It is a technically beautiful but hitherto little appreciated paper, extending to a regularly varying (and hence much more general) context, certain results of G. H. Hardy. It was to play a central role in the still to be completed final collaboration between Ranko and myself.

Thus began my correspondence with Ranko.

I spent the second half of 1968 in the Statistics Laboratory at Cambridge University where I met Nick Bingham who was beginning his research career with work related to the celebrated Karamata's Tauberian Theorem. I then spent the early months of 1969 at Imperial College, London, where Professor G.E.H. (Harry) Reuter was interested in the same questions related to regularly varying functions as I was, and in my interaction with, as he kindly and jokingly put it, "the three terrible Serbs". Much of that 1968–1969 Study Leave from the Australian National University was spent on regular variation.

My first joint paper with Ranko was motivated by a problem on explicit simple inversion for $f(t)$ of an asymptotic expression $f(x)L^\alpha(f(x)) \sim Ct^\beta$, $x \rightarrow \infty$ for $\beta > 0$, $C > 0$, and α real, within the context of a critical branching process. The problem was resolved primarily by a sufficient condition on $L(x)$ that $(L(\lambda_0 x)/L(x) - 1) \log L(x) \rightarrow 0$ as $x \rightarrow \infty$, for a fixed $\lambda_0 > 1$, due largely to Ranko's intimate knowledge of regularly varying function theory, and, in particular, of the existence of what later came to be called Karamata's Iteration Theorem.

A secondary aim of the paper was to present in English and in a well-known journal the brief history (in Section 1), and a complete presentation of the two fundamental theorems (in Section 4), of regularly varying function theory. The paper, Bojanić and Seneta (1971), was submitted through N. G. de Bruijn, one of the key figures in the post-war history of the theory, received March 20, 1970. The structure and exposition was very much, as in both of our joint papers, Ranko "doing it his way".

The purpose of our second paper, Bojanić and Seneta (1973), was to develop fully a theory of regularly varying sequences $(c(n), n = 0, 1, 2, \dots)$ from a definition of such sequences of Karamata, and hence in the Karamata sense, paralleling the theory of regularly varying functions. A definition of regularly varying sequences had been given by Galambos and Seneta (1973), so Ranko's motivation was to show that the two definitions were equivalent. A key to the equivalence was a result of Rada Higgins (1974), at the time a Ph.D. student of Ranko: that

if $(c(n), n = 0, 1, 2, \dots)$ is a regularly varying sequence according to the Karamata definition, then $c(n+1)/c(n) \rightarrow 1$ as $n \rightarrow \infty$. This second paper would also play a role in our final collaboration.

I met Ranko during my only visit to Ohio State early in the 1988–89 academic year while based at the University of Virginia, Charlottesville. He liked to communicate verbally, and we spent much of the time with Ranko recollecting the history of regular variation of the strong Yugoslav School founded by Karamata. We spent time with his friend from their student days in Belgrade from 1948, Bogdan Baishanski. And I still have a picture in my mind of Ranko feeding the squirrels on campus. At the airport in Columbus, Ohio, he sketched for me on the back of a large envelope the component steps of Karamata's proof of what is now called Karamata's Tauberian Theorem, and described Landau's intervention towards its publication in the *Mathematische Zeitschrift*. He looked on me and Nick Bingham as direct inheritors of the Karamata tradition, and passed onto us his Karamata relics. I inherited Karamata's copy of Pólya and Szegő's book, which had inspired the young Karamata to create the concept of a slowly varying function as a generalization of the iterated logarithm.

On June 1–10, 1989, there was an International Workshop in Analysis and its Applications in Kupari-Dubrovnik, Yugoslavia, a country then in the early days of transition. The Principal Theme was Karamata's Regular Variation, and I was able to attend, directly from Charlottesville. All the great and good of regular variation had been gathered there, including, of course, Ranko. There were papers, titled "Remembering Jovan Karamata", I and II, respectively by Aljančić & Tomić, and Bajshanski & Bojanic, in a session chaired by S.B. Stechkin, one of the eminent Russian participants. These included S. M. Nikolskii and V. M. Zolotarev. Following a presentation by one of the Russians which involved an outdated definition of regular variation, Ranko told me of the many discussions between Karamata and his students regarding what was the proper definition of a slowly varying function at infinity. They decided on the now classical definition: a function $L(x)$, $x > A$ defined and positive for fixed $A > 0$, is said to be slowly varying at infinity if it satisfies the condition $L(\lambda x)/L(x) \rightarrow 1$ as $x \rightarrow \infty$, for each $\lambda > 0$. The evenings of the conference were spent in amiable interaction over drinks. Some distance away there was music: an outdoor concert, nightly. I remember Ranko drifting away from us one night to sit watching people dancing the "kolo". That part of Yugoslavia, Dalmatia, was essentially Croatian, and our Yugoslav-born colleagues, all Serbians, seemed surprised.

In two email messages of March, 2009, Ranko wrote:

As you can see, I am still alive at 84. As far as my health is concerned, I am OK (with generous help of modern medical prescription). Once a week I have lunch with Bogdan ... I stopped doing mathematics in the Summer of 1995 when I retired. The main events that influenced this were the deaths of my two friends and collaborators Aljancic and Tomic. Mathematics made sense as long as I could share it with them, discuss proofs and discover together new results. In retirement, walks in the park with my dog Austin replaced mathematics. Internet took its share too. I would still like to see your paper. Please send two reprints (one for Bogdan) ...

I had written in a preceding email to Ranko that a condition in Bojanic and Seneta (1971) had given rise to a condition recently found to be necessary and sufficient (!) for a (probabilistic) Weak Law of Large Numbers by Csörgő and Simons (2008). These authors had called it the Bojanic–Seneta condition.

Our last collaboration began with an email from me on July 7, 2011, asking Ranko whether anyone had found a necessary and sufficient condition for the asymptotic behaviour of the cosine

series in terms of the behaviour of the coefficients, parallelling the theory for the sine series in Aljančić, Bojanić and Tomić (1956) in terms of regular variation in the Karamata sense. The theory for both the sine and cosine series in terms of regular variation in the more restrictive Zygmund sense was known from Volume 1 of A. Zygmund's majestic book *Trigonometric Series* of 1959/1968. Ranko's interest and recollections were aroused. He reread that old 1956 paper and some related papers, and responded (13 July, 2011).

I see now, being 58 years wiser, that it would not be so simple to obtain results for the cosine series from Theorems 1 and 2 [for the sine series in our 1956 paper]. ... I remember that in 1957 Aljancic and I were invited to a conference in Varenna, Italy, where Zygmund was the principal speaker. He gave a series of talks on kernel functions. He was very much interested in the results of our French paper... At the time the second edition of his *Trigonometric Series* was being prepared for printing and he asked me to read printed sheets of Chapter V during the conference. In addition ... I ... made several suggestions ... which Zygmund adopted. Zygmund appreciated very much my help and he wrote very good letters of recommendation on my behalf to Chandrasekharan in Bombay and Szegő in Palo Alto. (Karamata did also write such letters to Chandrasekharan and Szegő.) This is how I came to the School of Mathematics of the Tata Institute of Fundamental Research in June of 1958 and to Stanford University.

Ranko had seized with enthusiasm the idea of a slowly varying function in the Zygmund sense, and generalized it to the concept of a quasi-monotone slowly varying function (Bojanic and Karamata (1963), Section 4), during an American grant-funded visit of Karamata to the USA.

This last collaboration of ours developed for a few months, then stopped. I received an email of 25 February 2012 that Ranko had gone to Paris to see his younger brother who was in hospital with lung cancer, arriving on September 8 at 6 am, only to find that his brother had died three hours earlier.

Slowly I recovered here, enjoying walks in the park with my dog Austin and watching the Australian Open and other tennis tournaments, but I feel part of me is gone with Aleksandar. Fortunately my mathematical part seems to be intact.

Our correspondence resumed briefly in March, 2013, still on the theory of cosine series. I prepared a long manuscript letter, and the correspondence revived in earnest on June 19, 2014 with his response. In late September there were some comments from him, concluding with "I am not quite well, I am writing this letter from the OSU hospital".

But he continued to work; and his interest now focused on the precise relation between the Karamata and Zygmund concepts of a slowly varying function. An email of October 1, 2014, had a related attachment, but additionally brought the grave news that cancerous nodules had been discovered in his body. "The physicians here at OSU will try to use radiation and chemotherapy ... If I do not survive [the] next few months, please use all of this for our joint paper, write an obituary for Ranko, ...". The next email of October 26 was much more positive: "I have finished a series of 10 radiation treatments ... I feel quite well, no pains or any other discomfort". Our academic correspondence continued, and on December 27, 2014, he wrote:

I am still very much OK. The doctors have finally [been] able to diagnose all the problems I am facing ... I will enjoy again watching Novak Djokovic and Ana Ivanovic and several other Serbian players ... (at the Australian Open Tennis Tournament in Sydney). ... If a

game in Sydney starts at 4pm, here [in Columbus] it will be exactly midnight. I do not think I will be able to do any mathematics then, I will do it later.

But Ranko's condition deteriorated, and I contacted Paul Nevai after a time, as Ranko had advised me to do. On April 25, 2015, there was a brief note from Ranko to advise me that he was "still alive" and was still reading my draft joint paper.

There was a long silence again, during which I completed a typescript for submission of our joint paper by late 2015. A motivation for this had also been my correspondence in the interim with Gennady Samorodnitsky at Cornell, who was preparing a book (now published by Springer in 2016), which contained a detailed study of the cosine series from a Zygmund-like approach and a probabilistic standpoint.

Before formally submitting our paper, I emailed it to Ranko as an attachment. He was eventually well enough for our correspondence to resume for an intense few weeks, beginning with a long email from him on December 28, 2015. It carried strong themes of our earlier work on regularly varying sequences and of Zygmund's slowly varying functions, but refocusing on a proof of a cosine series version of the Aljančić, Bojanić, and Tomić (1956) results.

Basically, Ranko now wanted to do the paper "his way", as in our two former collaborations, and by his email of January 25, 2016, had sketched out the way he would do it if I agreed, and was ready to type it up formally. I received a last email from him February 7, 2016. Zita Divis wrote on April 11, 2016, that she had spoken with Ranko. "He said he was hoping to send you the work within a few weeks; certainly before mid-May". Sadly, that never happened. When I next wrote to Zita, early in the morning, Columbus time, of February 21, 2017, she advised that Ranko was on his death bed, and the next email advised his death that morning. So passed on the last of the brilliant Aljančić, Bojanić, and Tomić (1956) trio. It marks for me the end of an era, and another sad loss of a good friend.

But I do have all those emails from Ranko from our last intensive burst of collaboration in December 2015/January 2016, and our joint paper is well on its way to completion, quite substantially "his way".

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József & Cili Szabados in January of 1989 at Texas A&M in College Station, Texas.

József Szabados. I met Ranko Bojanic for the first time in 1969 in Budapest, at the Conference on Constructive Theory of Functions. He gave a talk on the ever so popular subject of Hermite–Fejér interpolation. It was Géza Freud who was in close contact with him, and he initiated my invitation to Columbus. (At that time Freud still had good relations with most of the Hungarian mathematicians.) So next year I made my first visit to the US. Ranko (together with his colleagues Zita and Bohus Divis) was an excellent host. He provided all the help which was needed for an inexperienced newcomer, including lodging, banking, teaching, etc. He even helped me to buy a black and white TV which he redeemed (at the same price) when I left Columbus, increasing the number of TVs he owned.

In the Fall of 1970, together with his former student Ron DeVore we traveled to College Park, Maryland to attend a conference organized in honor of Joseph Walsh’s 75th birthday. Thanks to Ranko, this gave me a unique opportunity to talk to Walsh about rational approximation of functions which was a hot topic at that time, initiated by the classic result of Donald Newman.

My next visit to Columbus was in the summer of 1974. I gave a talk at the Math. Department. I was surprised that after I finished talking, there were no comments, nor questions; people, including Ranko started disappearing from the room. Later I learned from Ranko that he was rushing to his office where there was a TV set, and he (and others) were eager to watch Nixon’s resignation speech (apparently more important than my talk...) Ranko was an experienced handyman, being interested not only in TVs, but anything electronic. I recall him showing me his first Texas Instrument calculator which was capable of performing the four basic arithmetic operations, and it cost him \$200. His favorite joke was to ask, Who is responsible for the oil crisis? He typed 11345 in the calculator, which after having turned upside down showed “Shell”. This was in August 1974, and we flew together to Madison, Wisconsin to attend the first US–Hungarian workshop on approximation theory, organized by Richard Askey. Ranko did not particularly enjoy flying, but he knew how to overcome anxiety by purchasing some pleasant “medication” at the airport.

The last time I met Ranko was in 2006, when I was visiting Columbus at the invitation of Paul Nevai. Ranko attended my talk, and we discussed some mathematics of his interest.

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